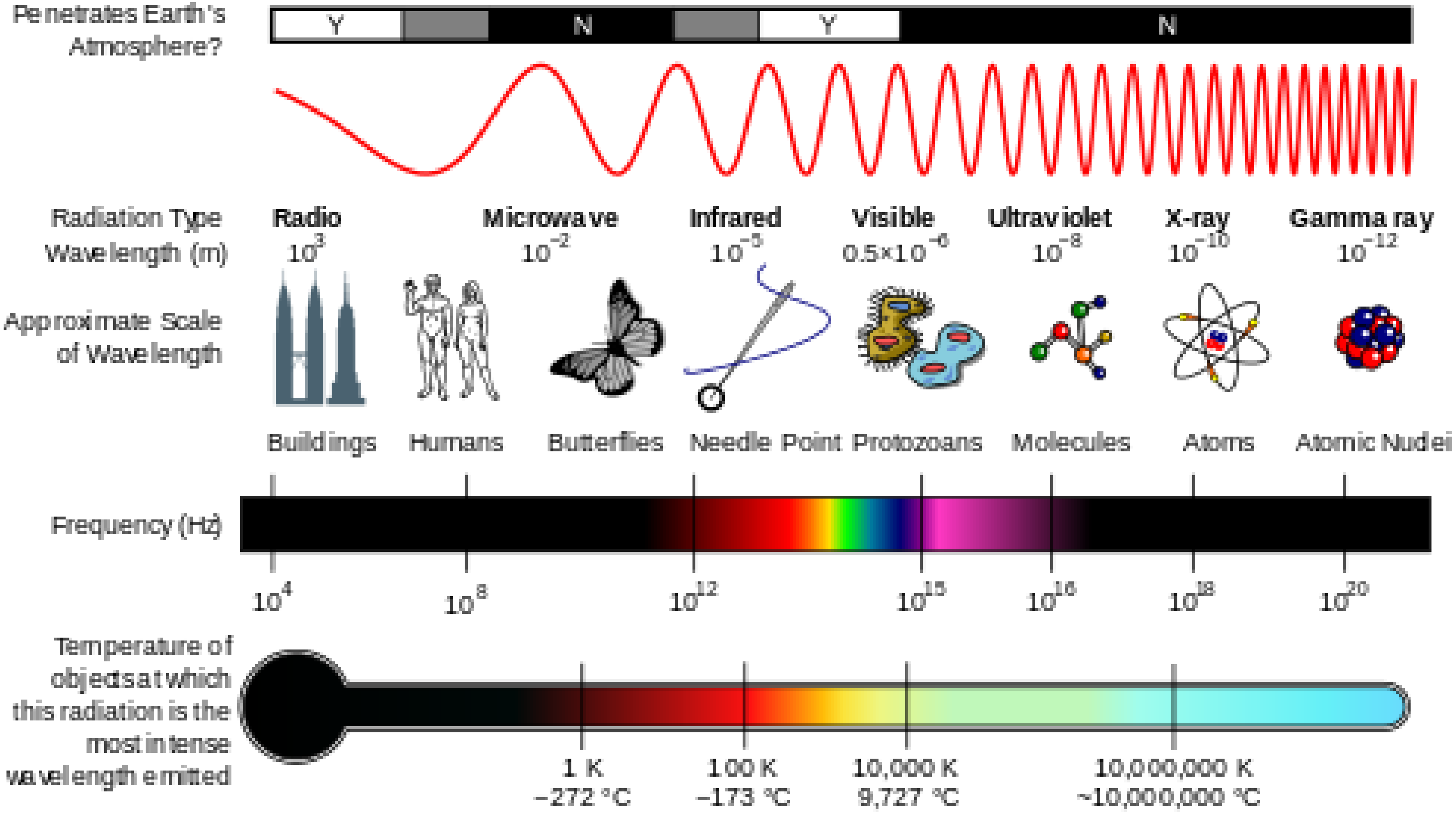
Maxwell’s equations

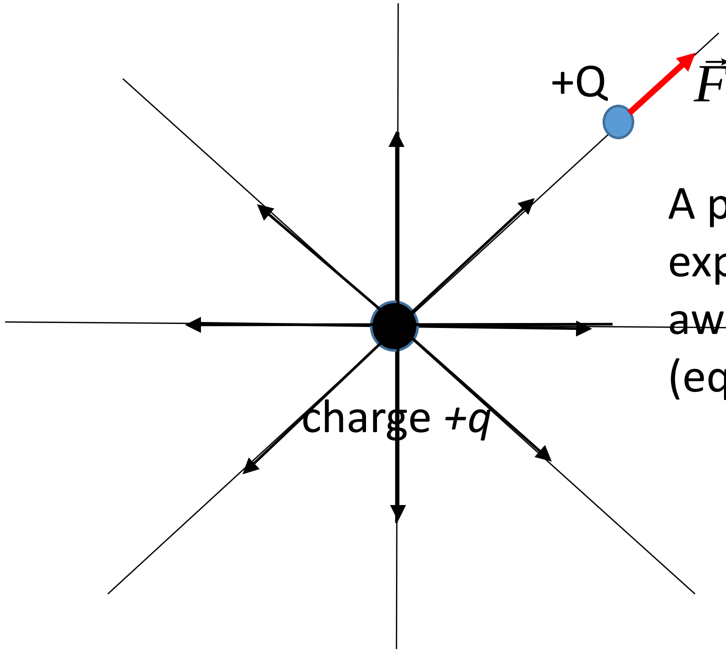
- Maxwell’s equations describe how electric and magnetic fields behave in the presence of charges and currents and the relationship between electric and magnetic fields.
- They unify the description of electric and magnetic fields as originating from a common phenomenon.
- They constitute one of the milestones in the history of theoretical physics, along with Newton’s laws of motion, Einstein’s relativity theory, and quantum mechanics.
- They predict the existence of ***electromagnetic waves*** and provide a unified understanding of the origin of the various forms of electromagnetic waves, from radio waves to visible light and gamma rays.

Electromagnetic spectrum



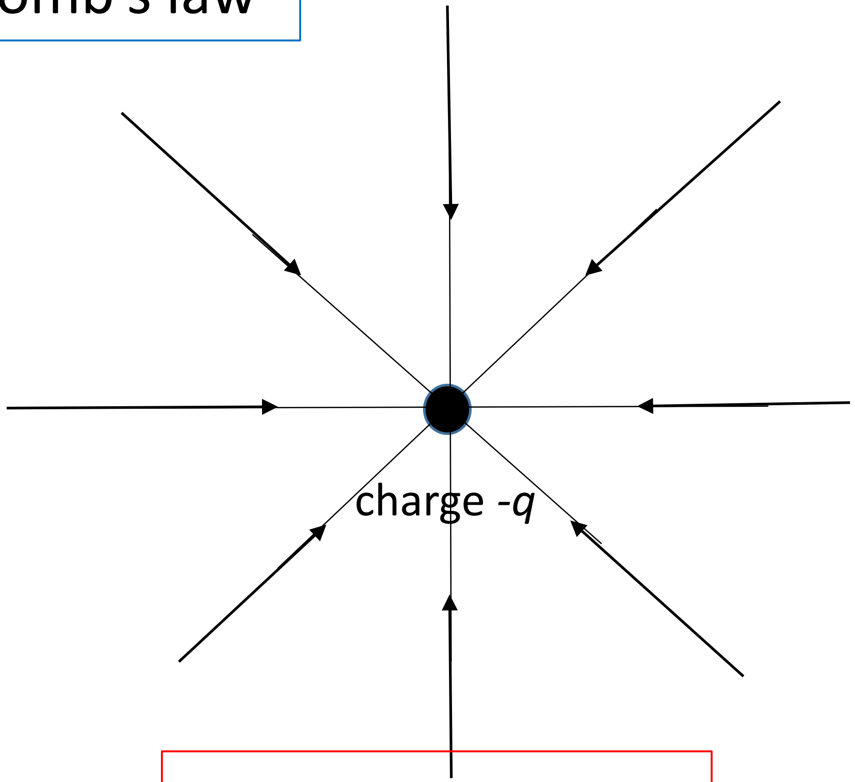
(Picture taken from Wikipedia: Electromagnetic spectrum - Wikipedia)

Maxwell's 1st equation: Coulomb's law



$$\vec{F}(\vec{r}) = \frac{qQ}{4\pi\varepsilon_0 r^2}\vec{e}_r$$

A positive charge +*Q* experiences a radial force away from the charge *q* (equal charges repel each other)



$$\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r^2}\vec{e}_r$$

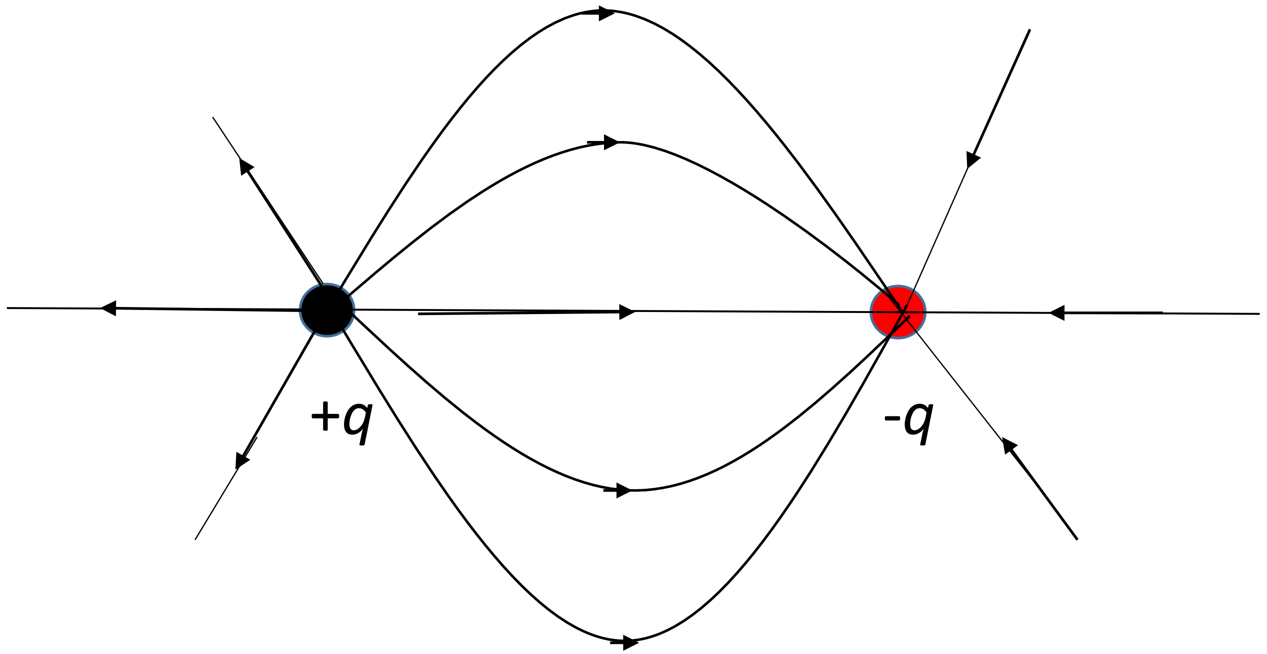
$$\vec{E}(\vec{r}) = -\frac{q}{4\pi\varepsilon_0 r^2}\vec{e}_r$$

The electric field lines from a *positive* point charge emanate radially *away* from the charge.

The electric field lines from a negative point charge converge radially *towards* the charge.

The line direction tells us the direction of force on a positive charge and the intensity (how dense the lines are) tells us the strength of the field.

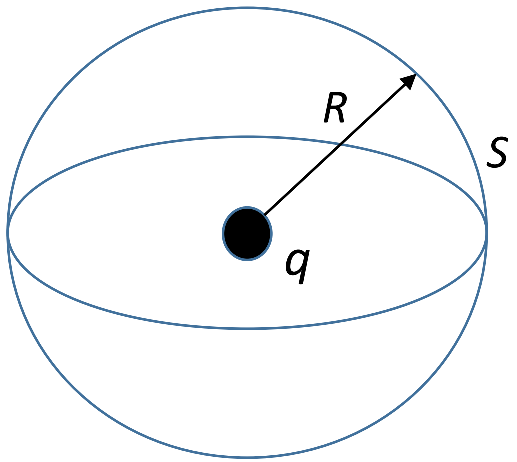
Electric field lines from a positive and a negative charge.



The electric field lines go out of the positive charge into the negative charge.

Derivation of Maxwell's 1st equation

1) Place a point charge q at the center of a sphere of radius R:



2) Calculate the surface integral of the electric field lines through the surface of the sphere:

$$\int_{S(V)} d\vec{S} \cdot \vec{E}(\vec{r}) = \int_{S(V)} dS\, \vec{e}_r \cdot \left(\frac{q}{4\pi\varepsilon_0 R^2} \vec{e}_r \right)$$

$$= \frac{q}{4\pi\varepsilon_0 R^2} \int_{S(V)} dS$$

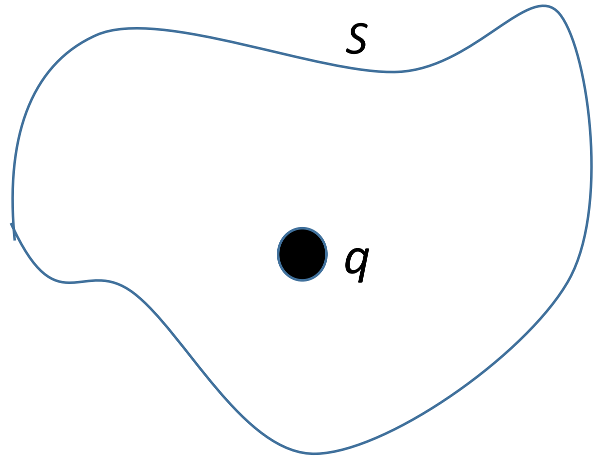
$$= \frac{q}{4\pi\varepsilon_0 R^2} \times 4\pi R^2 = \frac{q}{\varepsilon_0}$$

independent of R

Maxwell's 1st equation (integral form):

$$\int_{S(V)} d\vec{S} \cdot \vec{E}(\vec{r}) = \sum_i \frac{q_i}{\varepsilon_0}$$

True for ***any surface*** enclosing the charges



The equation also applies to this surface or any other surface enclosing the charge q.

Maxwell's 1st equation (differential form):

charge density = charge per unit volume

$$\int_{S(V)} d\vec{S} \cdot \vec{E}(\vec{r}) = \frac{1}{\varepsilon_0} \sum_i q_i = \frac{1}{\varepsilon_0} \int_{V(S)} dV \, \rho(\vec{r})$$

amount of charge
in a little volume *dV*

Use Gauss formula:

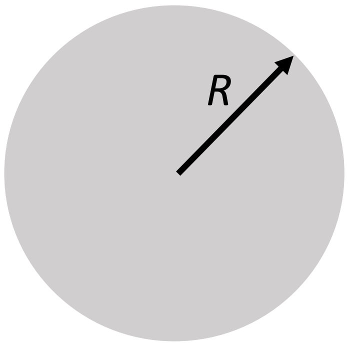
$$\int_{S(V)} d\vec{S} \cdot \vec{E}(\vec{r}) = \int_{V(S)} dV \left(\nabla \cdot \vec{E}(\vec{r}) \right) = \frac{1}{\varepsilon_0} \int_{V(S)} dV \, \rho(\vec{r})$$

$$\nabla \cdot \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

Maxwell's 1st equation (differential form)
or also known as Gauss law

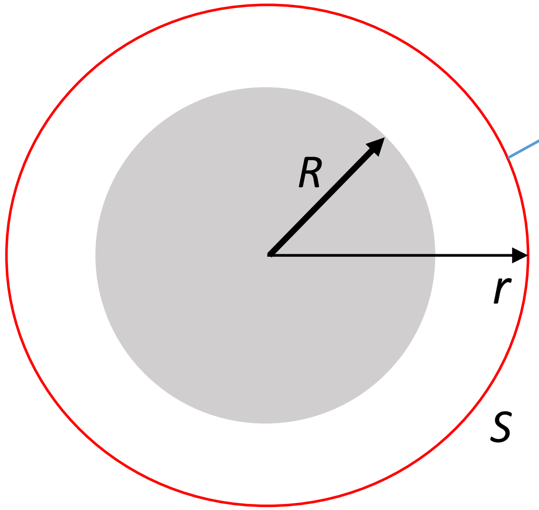
Example of the use of Maxwell's 1st equation

Consider a sphere of radius *R* containing a *uniform charge distribution* of density *ρ*.
We wish to figure out the electric field at a radial distance *r* from the center of the sphere when
$r > R$ and $r < R$



uniform charge density *ρ*

The case when $r > R$:



$$\vec{E}(\vec{r}) = E(r)\vec{e}_r$$

Total charge inside the red sphere:

$$Q = \int_{V(S)} dV \rho(\vec{r}) = \rho \int_{V(S)} dV = \frac{4\pi}{3} R^3 \rho$$

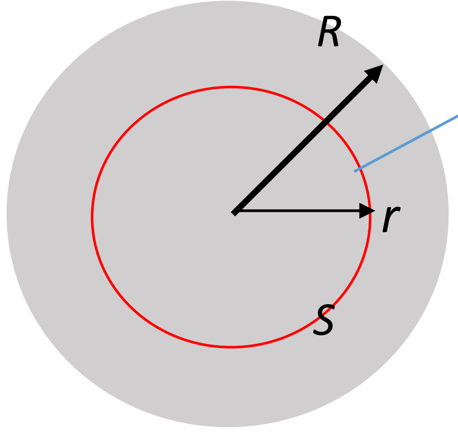
Maxwell's 1st:

$$\int_{S(V)} d\vec{S} \cdot \vec{E}(\vec{r}) = \int_{S(V)} dS \vec{e}_r \cdot [E(r)\vec{e}_r] = E(r) \int_{S(V)} dS = E(r) 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

It is the same as the electric field from a charge *Q* located at the center of the sphere!

The case when $r < R$:



$$\vec{E}(\vec{r}) = E(r)\vec{e}_r$$

Total charge enclosed by the red sphere:

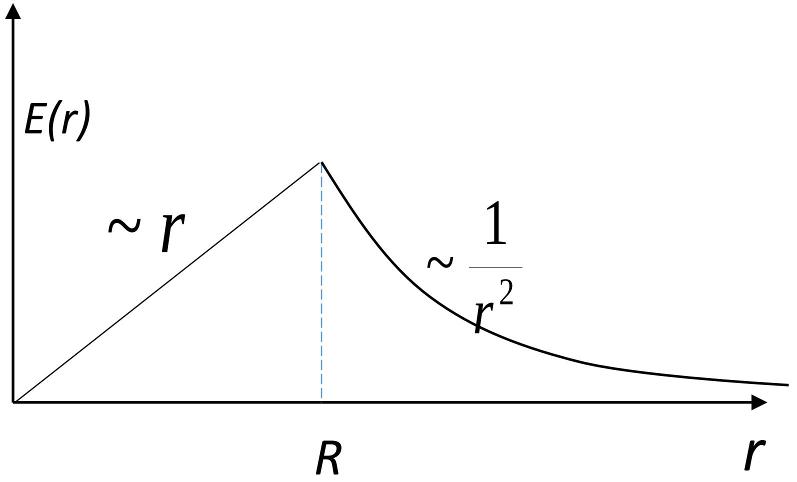
$$q(r) = \int_{V(S)} dV \rho(\vec{r}) = \rho \int_{V(S)} dV = \frac{4\pi}{3} r^3 \rho = \frac{r^3}{R^3} Q \qquad Q = \frac{4\pi}{3} R^3 \rho$$

Maxwell's 1st:

$$\int_{S(V)} d\vec{S} \cdot \vec{E}(\vec{r}) = \int_{S(V)} dS \vec{e}_r \cdot [E(r)\vec{e}_r] = E(r) \int_{S(V)} dS = E(r) 4\pi r^2 = \frac{q(r)}{\varepsilon_0}$$

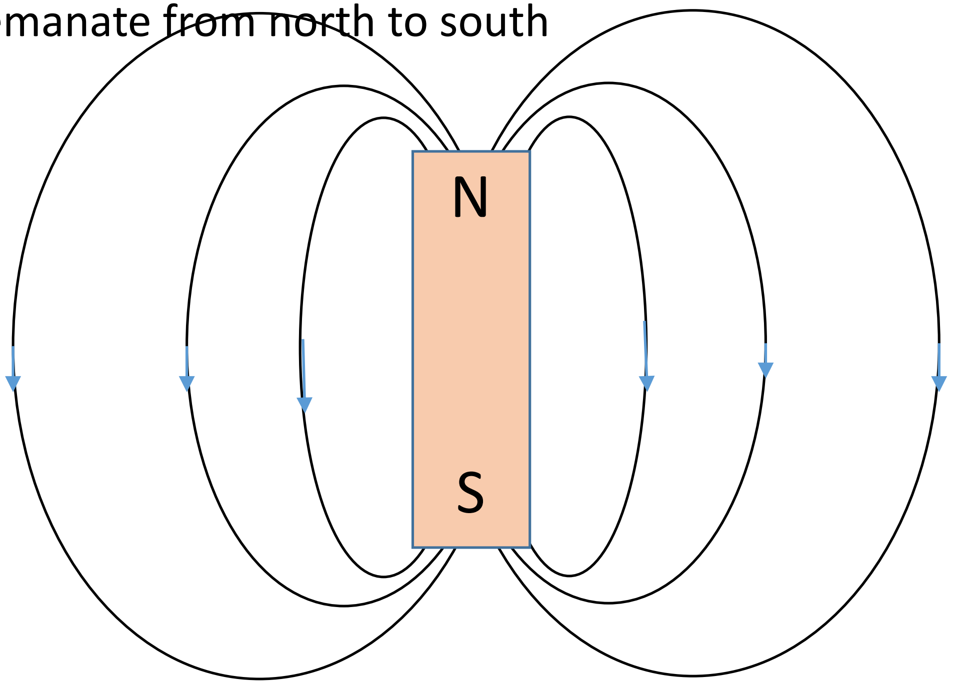
$$E(r) = \frac{q(r)}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 R^3} r$$

It is linear in *r* !



Maxwell's 2nd equation: No magnetic charge

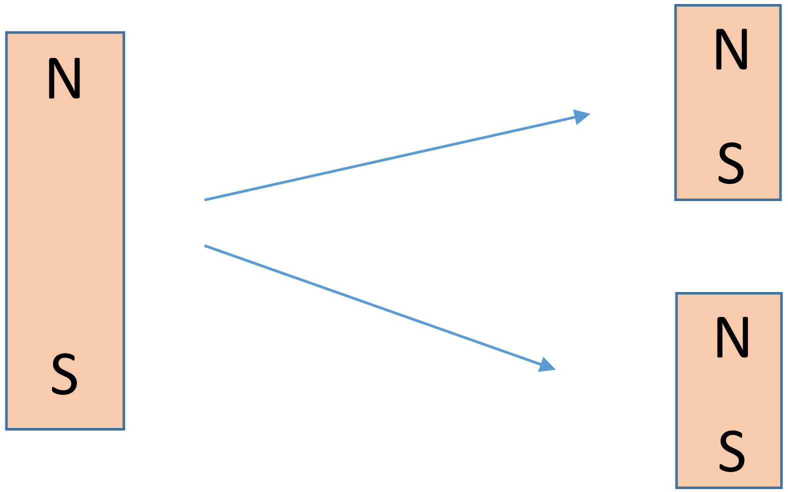
Magnetic field lines of a magnet emanate from north to south



$$\int_{S(V)} d\vec{S}\cdot\vec{B}(\vec{r}) = \int_{V(S)} dV\ \nabla\cdot\vec{B}(\vec{r}) = 0$$

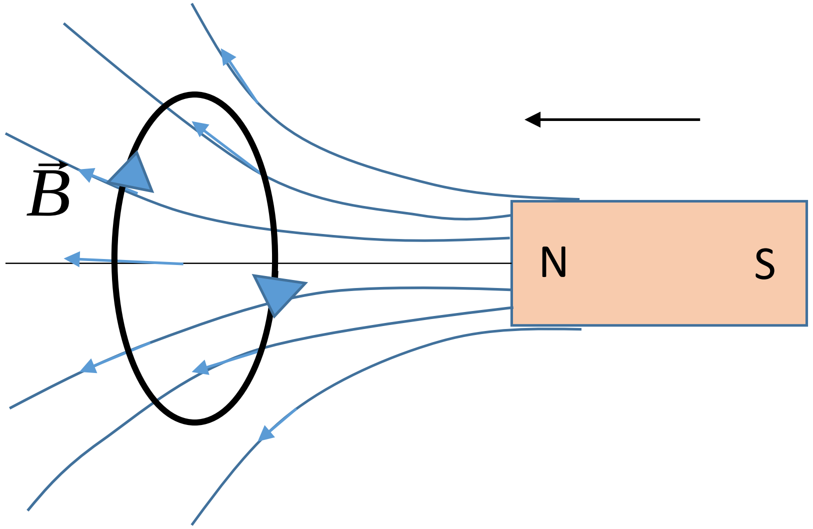
no magnetic charge

$$\nabla\cdot\vec{B}(\vec{r}) = 0$$



If a magnet is cut into two parts we get two magnets!
The North and the South cannot be separated.

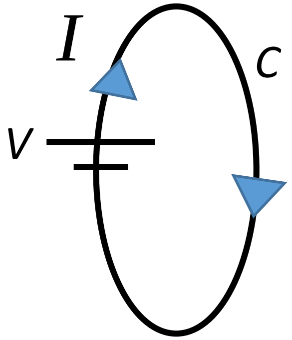
Maxwell's 3rd equation: Faraday's law



(seen from the left side)

A magnet moving towards a conducting loop induces current in the loop in the direction shown.

If the magnet moves away from the loop the induced current flows in the opposite direction.



Faraday's law:

$$V = -\frac{d\Phi}{dt}$$

magnetic flux through the loop C:

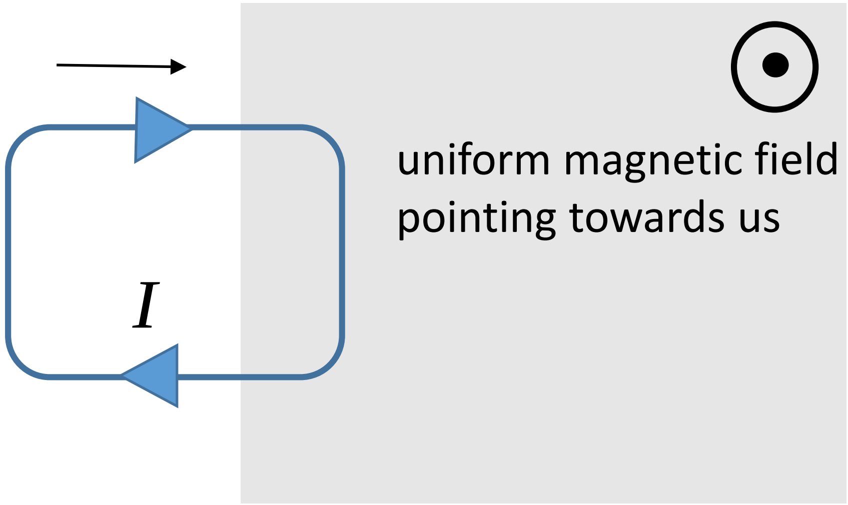
$$\Phi = \int_{S(C)} d\vec{S} \cdot \vec{B}(\vec{r})$$

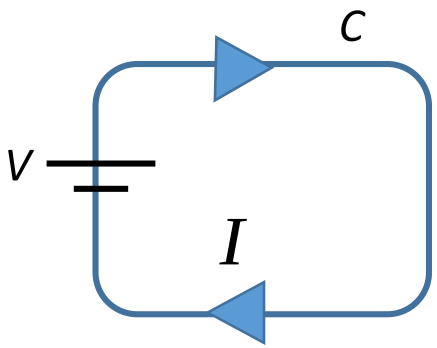
The crucial point is that the magnetic flux through the loop *changes in time*. If the flux is static, there is no current induced.

Induced voltage V is equal to the rate of change of magnetic flux through the loop

When applying Faraday's law, it is very important to define the circuit.

Maxwell's 3rd equation: Faraday's law





Faraday's law:

$$V = -\frac{d\Phi}{dt}$$

Induced voltage V is equal to the rate of change of magnetic flux through the loop

Current in a conducting loop is induced when the magnetic flux through the loop is increased or decreased.

magnetic flux through the loop C:

$$\Phi = \int_{S(C)} d\vec{S} \cdot \vec{B}(\vec{r})$$

The crucial point is that the magnetic flux through the loop *changes in time*.
If the flux is static, there is no current induced.

When applying Faraday's law, it is very important to define the circuit.

Derivation of Maxwell's 3rd equation

Faraday's law: $$V = -\frac{d\Phi}{dt} = -\int_{S(C)} d\vec{S} \cdot \frac{\partial \vec{B}(\vec{r})}{\partial t}$$

$$V = \oint_C d\vec{r} \cdot \vec{E}(\vec{r})$$

A voltage in a circuit or loop *C* is the work done in bringing a positive unit charge around the loop.

$$= \int_{S(C)} d\vec{S} \cdot \left(\nabla \times \vec{E}(\vec{r})\right)$$

from Stokes formula

$$\int_{S(C)} d\vec{S} \cdot \left(\nabla \times \vec{E}(\vec{r})\right) = -\int_{S(C)} d\vec{S} \cdot \frac{\partial \vec{B}(\vec{r})}{\partial t}$$

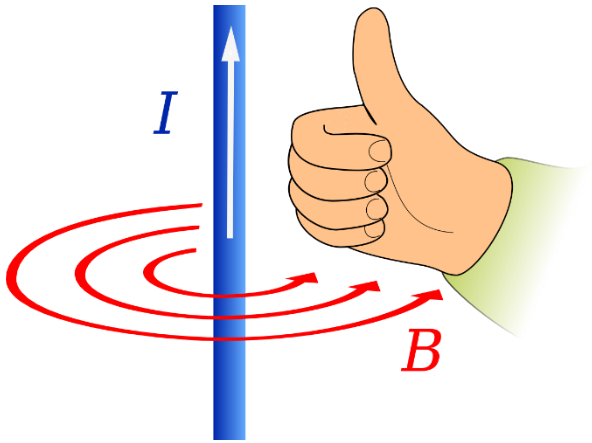
$$\nabla \times \vec{E}(\vec{r}) = -\frac{\partial \vec{B}(\vec{r})}{\partial t}$$

Maxwell's 3rd (differential form)

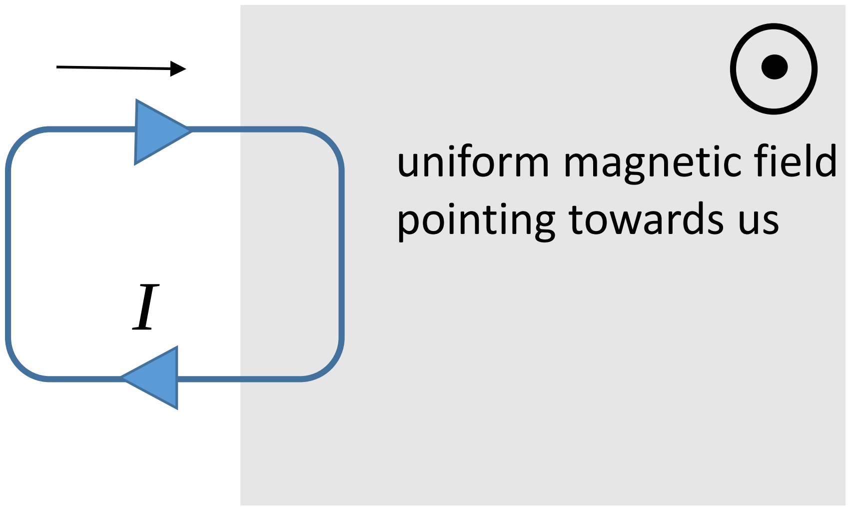
$$V = -\frac{d\Phi}{dt}$$

Lenz law: direction of induced current

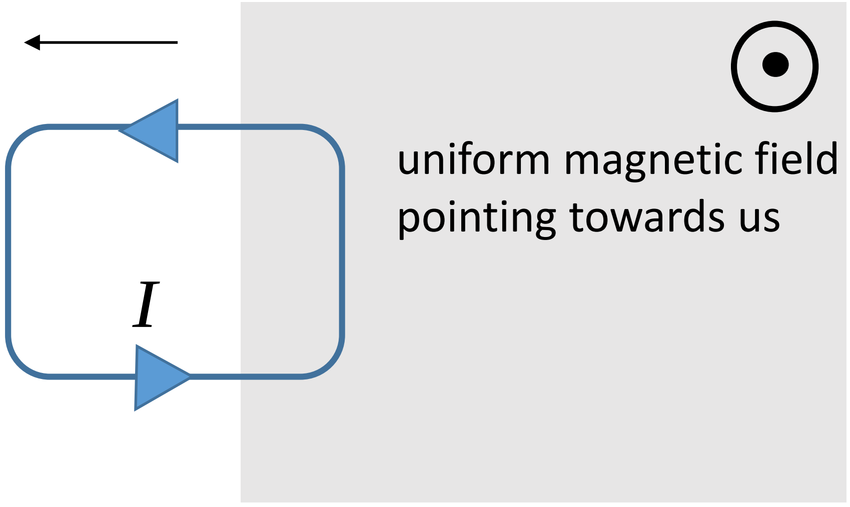
Right-hand rule:



(Picture from Wikipedia: Right-hand rule - Wikipedia)

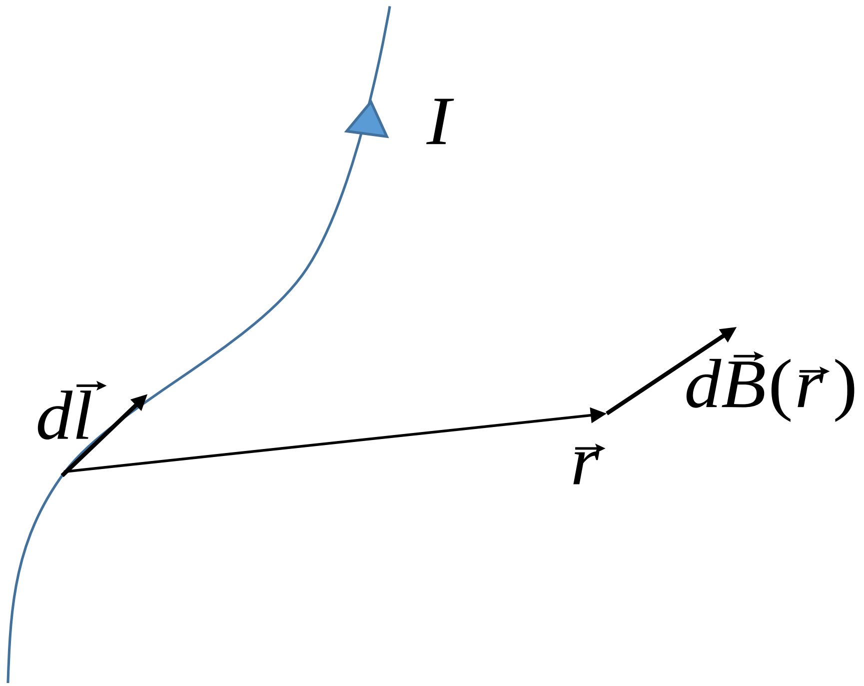


The magnetic field produced by the induced current *tries to keep the magnetic flux through the loop constant*, i.e., away from us to reduce the increasing flux.



Here, the magnetic field produced by the induced current is towards us in order to increase the decreasing flux, to keep the flux constant.

Biot-Savart law: the analogue of Coulomb's law for current



$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

contribution to the magnetic field at ***r*** from a small current element $I d\vec{l}$

compare with $$d\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{e}_r$$

Ampere's law

$$2\pi r B(r) = \oint_{C(S)} d\vec{r} \cdot \vec{B}(\vec{r})$$

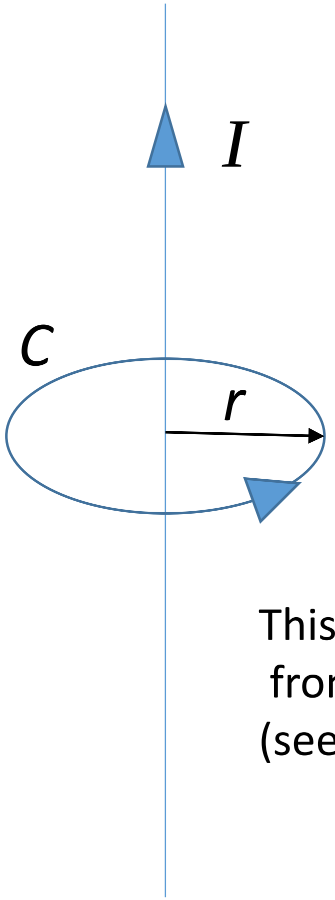
$$\oint_{C(S)} d\vec{r} \cdot \vec{B}(\vec{r}) = \mu_0 I$$

Ampere's law

$$\int_{S(C)} d\vec{S} \cdot \left(\nabla \times \vec{B}(\vec{r})\right) = \mu_0 \int_{S(C)} d\vec{S} \cdot \vec{j}(\vec{r}) \rightarrow \nabla \times \vec{B}(\vec{r}) = \mu_0 \vec{j}(\vec{r})$$

from Stokes

current density



$$B(r) = \frac{\mu_0 I}{2\pi r}$$

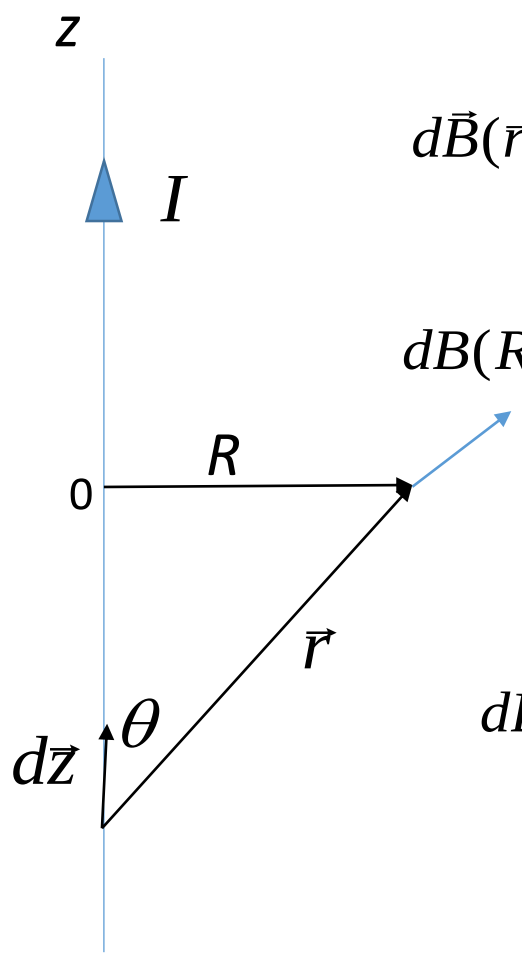
This result can be derived from Biot-Savart law (see next slide)

The line integral of *B* around a loop *C* is given by the current flowing through the surface enclosing the loop.

Note that *__the surface is arbitrary, as long as it encloses the loop C.__*

Ampere's law is valid for a *__constant current__* but it breaks down when the current changes in time.
The modification of Ampere's law leads to Maxwell's 4th equation.
It is a very important contribution from Maxwell,
which predicts the existence of electromagnetic waves.

$$B(R) = \frac{\mu_0 I}{2\pi R}$$ from Biot-Savart law



$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\frac{d\vec{z}\times\hat{r}}{r^2}$$

$$dB(R) = \frac{\mu_0 I}{4\pi}\frac{dz\sin\theta}{r^2}$$

$$\frac{R}{r} = \sin\theta \rightarrow \frac{1}{r^2} = \frac{\sin^2\theta}{R^2}$$

$$-z = R\cot\theta \rightarrow dz = \frac{R}{\sin^2\theta}d\theta$$

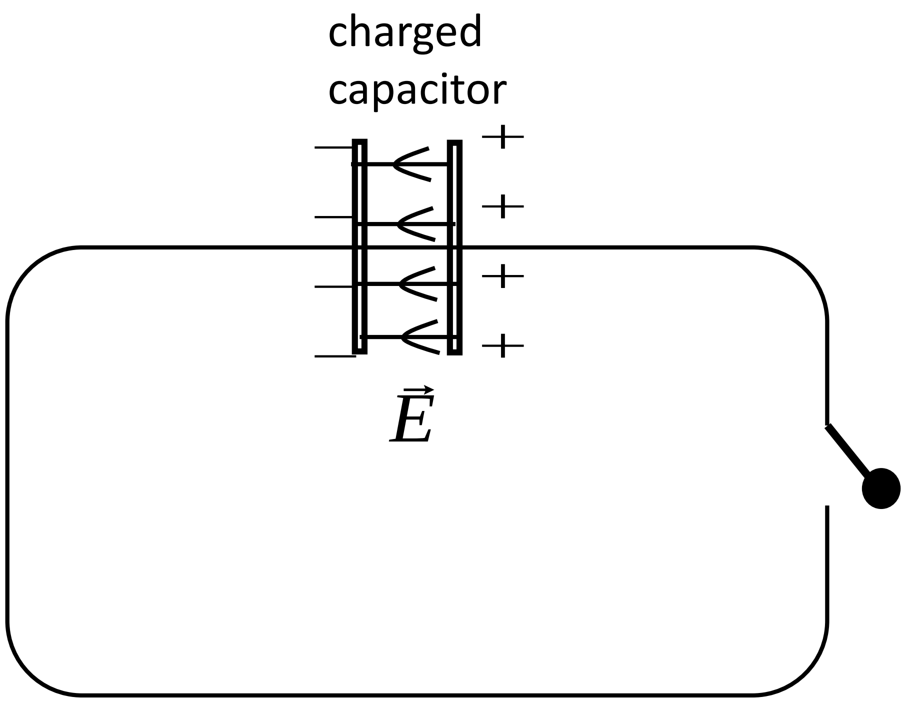
$$\frac{dz}{r^2} = \frac{d\theta}{R}$$

Integrate from $z = -\infty$ to 0,
which is equivalent to integrating from $\theta = 0$ to $\frac{\pi}{2}$

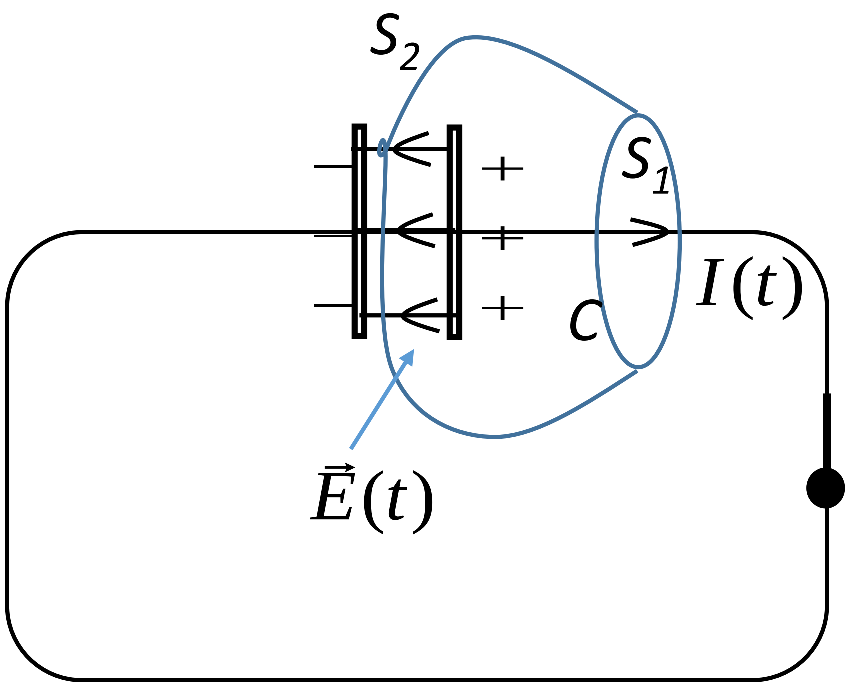
$$dB(R) = \frac{\mu_0 I}{4\pi}\frac{d\theta\sin\theta}{R} \rightarrow B(R) = \frac{\mu_0 I}{4\pi}2\int_0^{\frac{\pi}{2}} d\theta\frac{\sin\theta}{R} = \frac{\mu_0 I}{2\pi R}$$

(The factor of 2 arises because the contribution from $z = 0$ to ∞ is the same as the contribution from $z = -\infty$ to 0)

Fundamental problem with Ampere's law



Switch on the circuit →



Current flows around the circuit which decays with time as the charge in the capacitor is depleted.

According to Ampere's law,
the line integral of the magnetic field around the loop C is given by the current flowing through the surface enclosing the loop:

$$\oint_{C(S)} d\vec{r} \cdot \vec{B}(\vec{r}) = \mu_0 I$$

There is no current flowing through surface S_2.
Ampere's law is violated!

Notice that there is electric field piercing through S_2 which decreases with time.

What is missing in Ampere's law?

Consider the volume enclosed by the surface $S = S_1 + S_2$.

The current flowing out of the volume is given by $I(t) = -\dfrac{dQ(t)}{dt}$ ← charge in the capacitor

$$I(t) = \int_{S(V)} d\vec{S} \cdot \vec{j}(\vec{r},t)$$

$$\frac{d}{dt} Q(t) = \frac{d}{dt} \int_{V(S)} dV \; \rho(\vec{r},t)$$

$$\int_{S(V)} d\vec{S} \cdot \vec{j}(\vec{r},t) = -\int_{V(S)} dV \; \frac{d}{dt} \rho(\vec{r},t)$$

II (Gauss)

$$\int_{V(S)} dV \, \nabla \cdot \vec{j}(\vec{r},t)$$

$$\nabla \cdot \vec{j}(r,t) = -\frac{\partial \rho(\vec{r},t)}{\partial t}$$

Continuity equation
(conservation of charge)

According to Ampere's law $\nabla \times \vec{B}(\vec{r}) = \mu_0 \vec{j}(\vec{r})$

$$\underbrace{\nabla \cdot [\nabla \times \vec{B}(\vec{r})]} = \mu_0 \nabla \cdot \vec{j}(\vec{r}) = 0$$

= 0 (mathematical identity)

Continuity equation (fundamental in physics) is *not fulfilled*!

Maxwell's 4th equation

Modify Ampere's law as follows:

$$\nabla \times \vec{B} = \mu_0 \left(\vec{j} + \vec{X} \right)$$

such that the continuity equation is fulfilled:

$$\nabla \cdot (\nabla \times \vec{B}) = \mu_0 \nabla \cdot \left(\vec{j} + \vec{X} \right) = 0$$

$$\nabla \cdot \vec{X} = -\nabla \cdot \vec{j} = \frac{\partial \rho}{\partial t}$$

From Maxwell's 1st $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \rightarrow \frac{\partial \rho}{\partial t} = \varepsilon_0 \nabla \cdot \frac{\partial \vec{E}}{\partial t}$

$$\vec{X} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

(Sometimes called "displacement current")

$$\boxed{\nabla \times \vec{B} = \mu_0 \left(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)}$$

Maxwell's 4th

Maxwell's equations

Differential form	Stokes-Gauss	Integral form
$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$	Coulomb's law	$\int_{S(V)} d\vec{S} \cdot \vec{E} = \sum_i \frac{q_i}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$
$\nabla \cdot \vec{B} = 0$	No magnetic charge	$\int_{S(V)} d\vec{S} \cdot \vec{B} = 0$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Faraday's law	$\int_{C(S)} d\vec{r} \cdot \vec{E} = -\int_{S(C)} d\vec{S} \cdot \frac{\partial \vec{B}}{\partial t}$
$\nabla \times \vec{B} = \mu_0 \left(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$	Maxwell's contribution	$\oint_{C(S)} d\vec{r} \cdot \vec{B} = \mu_0 \varepsilon_0 \int_{S(C)} d\vec{S} \cdot \left(\frac{\vec{j}}{\varepsilon_0} + \frac{\partial \vec{E}}{\partial t} \right)$

E and ***B*** are ***inter-related*** when they change with time

Electromagnetic wave equations in vacuum

In vacuum there is no charge and no current so that Maxwell's equations become

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Consider taking a rotation on Maxwell's 3rd :

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

From Rule 2: $\nabla\left(\nabla \cdot \vec{E}\right) - \left(\nabla \cdot \nabla\right)\vec{E}$

$$\underbrace{\nabla\left(\nabla \cdot \vec{E}\right)}_{0} - \underbrace{\left(\nabla \cdot \nabla\right)\vec{E}}_{\nabla^2 \vec{E}}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Similarly, taking the rotation of Maxwell's 4th yields

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Wave equations in 3D for ***E*** and ***B***

Solutions to the wave equation

The wave equation in 1D has the general form $$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$ (A second-order differential equation has two independent solutions)

The solution is given by $$f = f_L(x+ct) + f_R(x-ct)$$

f_L and f_R are ***arbitrary functions.***

$f_L(x+ct)$ moves to the left; $f_R(x-ct)$ moves to the right, with speed c

Check that it is a solution to the wave equation. Let $y = x+ct$

Chain rule of differentiation:

$$\frac{\partial f_L}{\partial x} = \frac{\partial f_L}{\partial y}\frac{\partial y}{\partial x} = \frac{\partial f_L}{\partial y} \qquad \frac{\partial f_L}{\partial t} = \frac{\partial f_L}{\partial y}\frac{\partial y}{\partial t} = c\frac{\partial f_L}{\partial y}$$

$$\frac{\partial^2 f_L}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f_L}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f_L}{\partial y}\right)\frac{\partial y}{\partial x} = \frac{\partial^2 f_L}{\partial y^2}$$

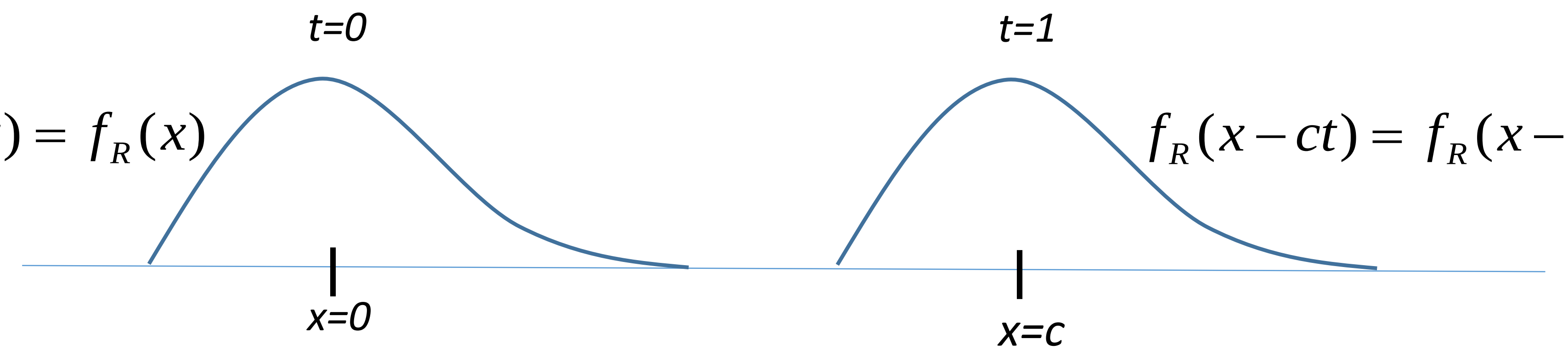
$$\frac{\partial^2 f_L}{\partial t^2} = c\frac{\partial}{\partial t}\left(\frac{\partial f_L}{\partial y}\right) = c\frac{\partial}{\partial y}\left(\frac{\partial f_L}{\partial y}\right)\frac{\partial y}{\partial t} = c^2\frac{\partial^2 f_L}{\partial y^2}$$

$$\frac{\partial^2 f_L}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f_L}{\partial t^2}$$

(We have not specified f_L so it is arbitrary)

Let us plot $f_R(x\text{-}ct)$ for $t=0$ and $t=1$:

$$f_R(x-ct) = f_R(x)$$



$$f_R(x-ct) = f_R(x-c)$$

In one second, the wave packet moves a distance c.
In other words, its speed is c.

Consider now the electromagnetic wave equation in 1D for E_x and assume that it depends only on x (in general E_x would depend on x, y, and z):

$$\frac{\partial^2 E_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

Comparison with the wave equation shows that:

$$c^2 = \frac{1}{\mu_0 \varepsilon_0} \rightarrow c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \, m/s$$

Maxwell not only predicted the existence of electromagnetic waves but also predicted the speed!

Harmonic solutions to the wave equation

$$f_R = A\exp[i(kx - \omega t)]$$

$$f_L = A\exp[i(-kx - \omega t)]$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

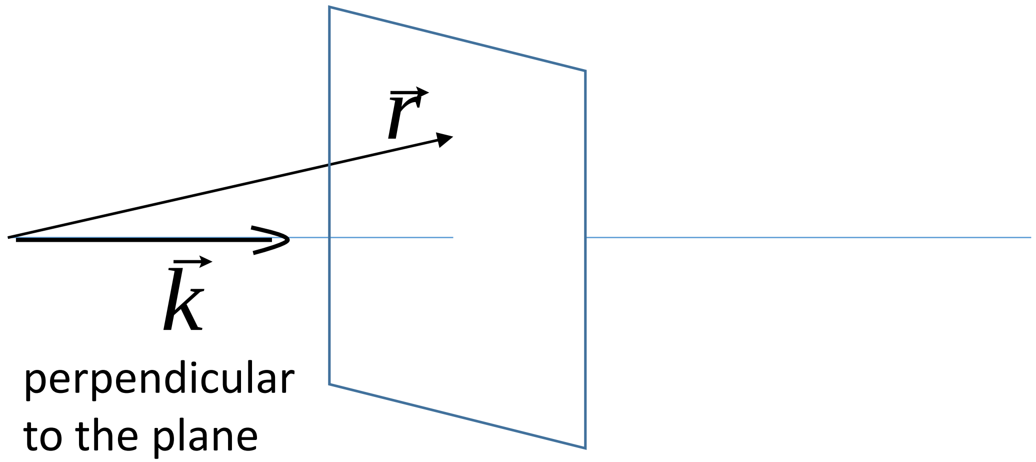
$$A(ik)^2 = \frac{1}{c^2}A(-i\omega)^2 \rightarrow k^2 = \frac{\omega^2}{c^2}$$

$$\boxed{\omega(k) = \pm ck}$$ dispersion relation

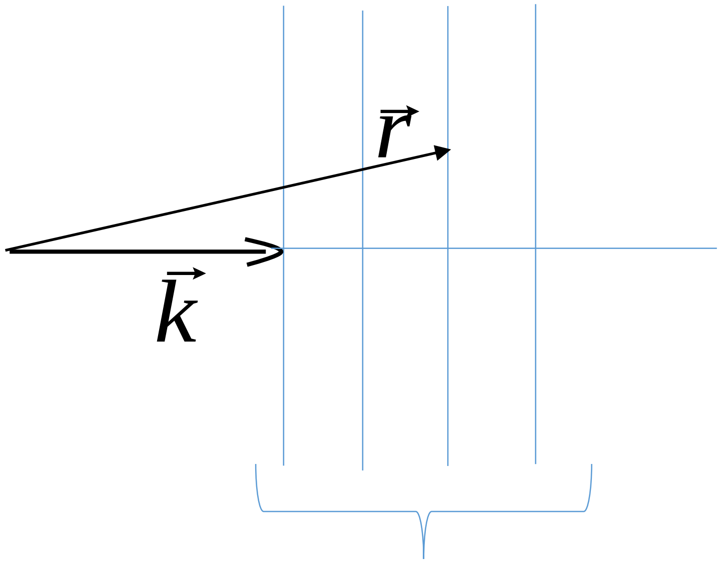
Plane-wave solutions to the wave equation in 3D

$$f(\vec{r},t) = A\exp\left[i(\vec{k}\cdot\vec{r}-\omega t)\right] = A\exp\left[i(k_x x + k_y y + k_z z - \omega t)\right]$$

phase



any point on the plane has the same phase



plane of equal phase
moves to the right

$$\vec{E}(\vec{r},t)=\vec{E}_0\exp\left[i(\vec{k}\cdot\vec{r}-\omega t)\right] \qquad \vec{E}_0=\left(E_{0x},E_{0y},E_{0z}\right)$$

$$\vec{B}(\vec{r},t)=\vec{B}_0\exp\left[i(\vec{k}\cdot\vec{r}-\omega t)\right] \qquad \vec{B}_0=\left(B_{0x},B_{0y},B_{0z}\right)$$

$$\nabla\cdot\vec{E}=0 \qquad i\vec{k}\cdot\vec{E}=0$$

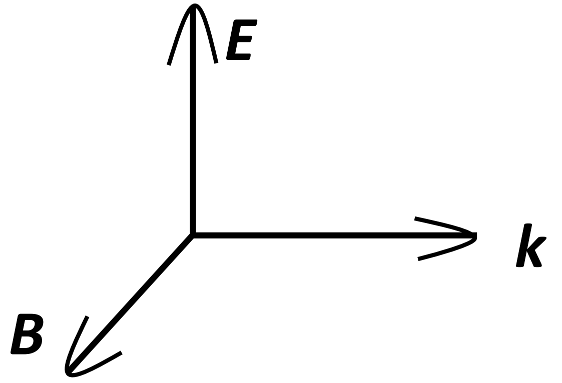
$$\nabla\cdot\vec{B}=0 \qquad i\vec{k}\cdot\vec{B}=0$$

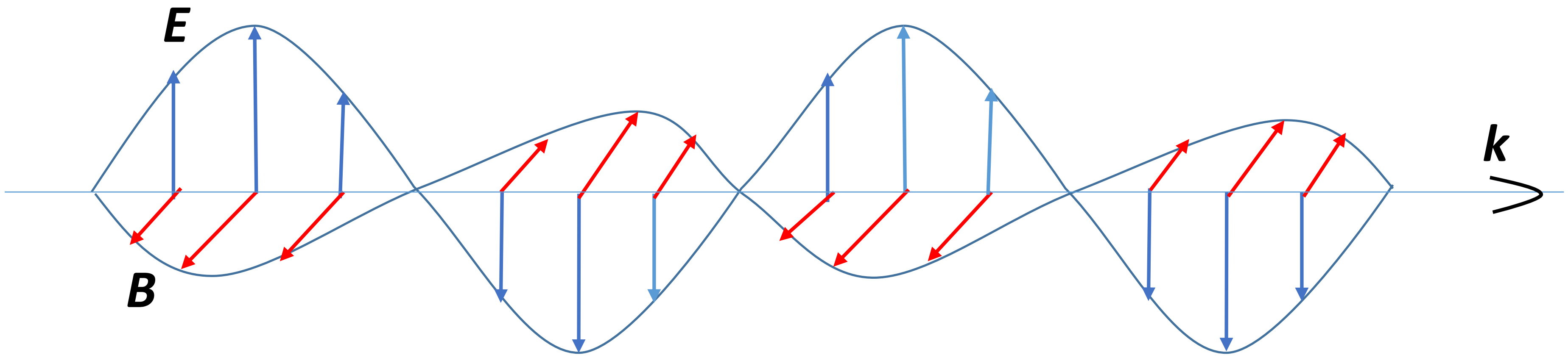
E and ***B*** are perpendicular to ***k***

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad i\vec{k}\times\vec{E}=i\omega\vec{B}$$

$$\nabla\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad i\vec{k}\times\vec{B}=\mu_0\varepsilon_0(-i\omega)\vec{E}$$

E is perpendicular to ***B***





$$\boxed{\nabla \cdot \vec{E}}$$

$$\vec{E}(\vec{r},t) = \vec{E}_0 \exp\left[i(\vec{k}\cdot\vec{r} - \omega t)\right]$$

$$E_x(\vec{r},t) = E_{0x} \exp\left[i(\underbrace{k_x x + k_y y + k_z z}_{\vec{k}\cdot\vec{r}} - \omega t)\right]$$

$$E_y(\vec{r},t) = E_{0y} \exp\left[i(k_x x + k_y y + k_z z - \omega t)\right]$$

$$E_z(\vec{r},t) = E_{0z} \exp\left[i(k_x x + k_y y + k_z z - \omega t)\right]$$

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= ik_x E_{0x} + ik_y E_{0y} + ik_z E_{0z}$$

$$= i\vec{k}\cdot\vec{E}$$

$$\frac{\partial \vec{E}}{\partial t} = -i\omega\vec{E}$$

$$\nabla \times \vec{E}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \vec{e}_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \vec{e}_y\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \vec{e}_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$= \vec{e}_x\left(ik_yE_z - ik_zE_y\right) + \vec{e}_y\left(ik_zE_x - ik_xE_z\right) + \vec{e}_z\left(ik_xE_y - ik_yE_x\right)$$

$$= i\vec{k} \times \vec{E}$$

For a plane wave (not in general):

$$\nabla \to i\vec{k}$$

$$\frac{\partial}{\partial t} \to -i\omega$$

Magnitudes of ***E*** a μT nd ***B***

$$i\vec{k}\times\vec{E} = i\omega\vec{B} \xrightarrow{\text{magnitudes}} |\vec{k}\times\vec{E}| = \omega|\vec{B}|$$

$$kE = \omega B$$ since ***k*** is perpendicular to ***E***

$$B = \frac{k}{\omega}E = cE$$

A typical value of *E* for a radio wave is *40 V/m* → *B = 0.1 μT* (very weak).
Compare with the earth's magnetic field of about *50 μT.*
A small bar magnet has about *0.01 T.*

The direction of the electric field is used to define the direction of light polarization.

Maxwell's equations in a transparent material

Still valid but the electric permittivity and magnetic permeability are modified depending on the material.

$$\varepsilon_0 \to \varepsilon = \varepsilon_r \varepsilon_0, \quad \mu_0 \to \mu = \mu_r \mu_0$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu \left(\vec{j} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right)$$

The speed of light in a material is given by $c_{mat} = \frac{1}{\sqrt{\varepsilon \mu}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$

Examples:

	ε_r
Air	1.00059
Water	80.1
Pyrex (glass)	4.7
Silicon	11.7

	μ_r
Iron (Fe)	5000
Neodymium magnet	1.05
Aluminium	1.000 022
Nickel	100