

Vecka 1

Kap. 1-3

* Komplexa tal

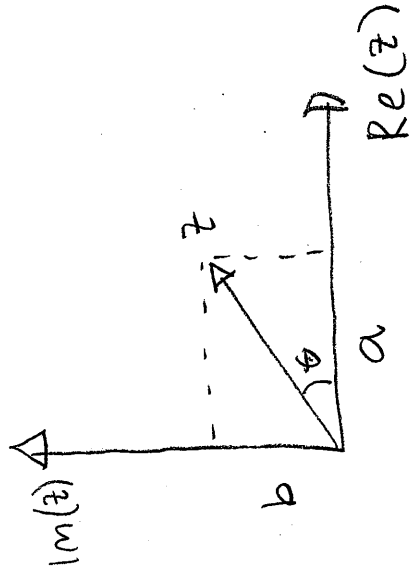
* Reella fourierserier

Komplexa tal

Eulers formel

$$e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

Polär form



$$z = a + ib$$

$$z = |z| e^{i\theta}$$

$$|z| = \sqrt{a^2 + b^2}$$

$$\theta = \arg(z) = \arctan(b/a)$$

Exempel

$$z = -1 \text{ på polär form}$$

$$z = 1 \cdot e^{i\pi} = e^{i\pi}$$

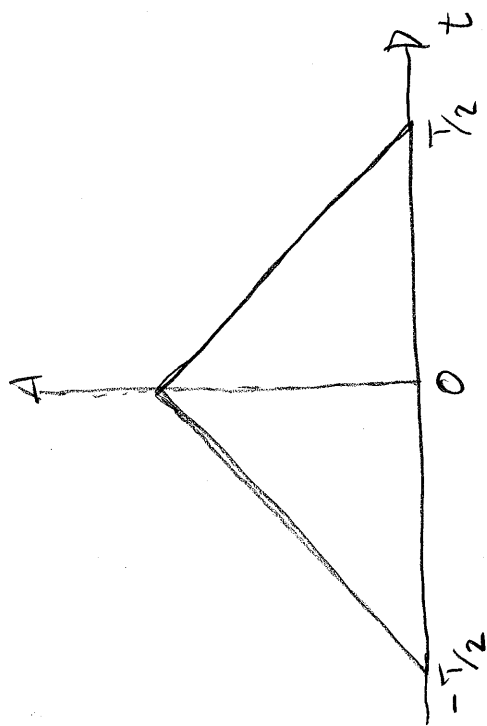
Reelle Fourierserie

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)), \quad \omega_0 = \frac{2\pi}{T}, \quad t \in \left[-\frac{T}{2}, \frac{T}{2}\right]$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} \cos(n\omega_0 t) f(t) dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} \sin(n\omega_0 t) f(t) dt$$

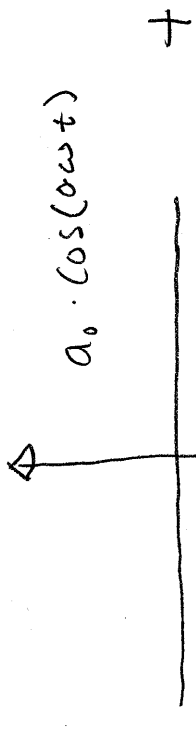
Example



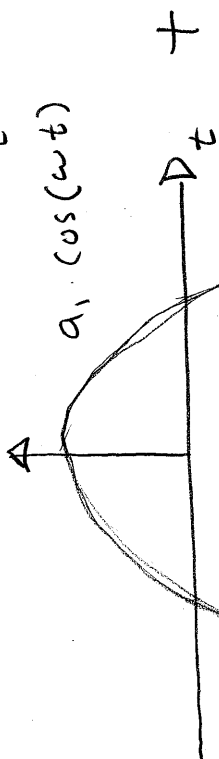
even function

$$\Rightarrow b_n = 0$$

$$a_0 = 1$$

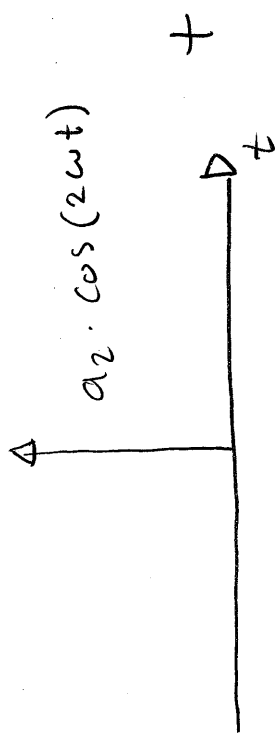


$$a_1 = \frac{4}{\pi^2}$$

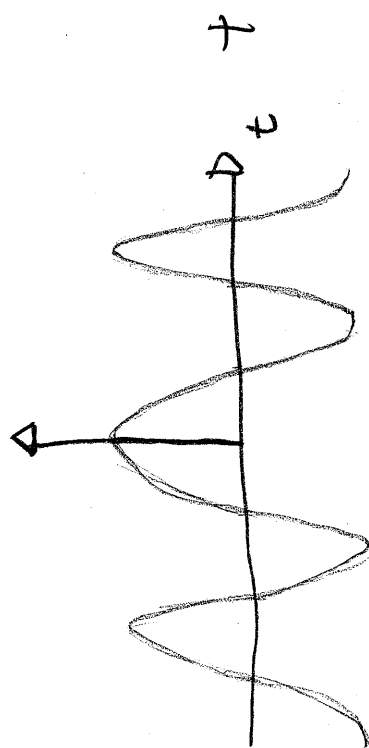


=

$$a_2 = 0$$



$$a_3 = \frac{4}{9\pi^2}$$



...

Härledning av formeln

Om $f(t)$ är en jämn funktion $\Rightarrow b_n = 0$

$$f(t) = \sum_{n=0}^{\infty} a_n \cdot \cos(n\omega_0 t)$$

g²ängs

båda sidor med $\cos(m\omega_0 t)$ & integrera

$$\int_{-T/2}^{T/2} \cos(m\omega_0 t) f(t) dt = \sum_{n=0}^{\infty} a_n \int_{-T/2}^{T/2} \cos(m\omega_0 t) \cos(n\omega_0 t) dt$$

δ_{nm} $T/2$

$\cos(n\omega_0 t)$ är
ortogonala p.f.
intervallet $[-T/2, T/2]$

$$\Rightarrow a_m = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(m\omega_0 t) dt$$

Vecka 2

Kap. 4-11

- * Den komplexa fourierserien
- * Fourierspektrat
- * Värmeledningsekvationen
- * fouriertransformen

Komplexa fourierserier

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega_0 t}$$

gångs med $e^{-im\omega_0 t}$ e integreras

$$\int_{-T/2}^{T/2} e^{-im\omega_0 t} f(t) dt = \sum_{n=-\infty}^{\infty} c_n$$

$$\underbrace{\int_{-T/2}^{T/2} e^{i(n-m)\omega_0 t} dt}_{\delta_{nm} T}$$

$$\Rightarrow c_m = \frac{1}{T} \int_{-T/2}^{T/2} e^{-im\omega_0 t} f(t) dt$$

Fourierspektret

Fourierspektret c_0, c_1, c_2, \dots är komplexa $f = 1$

Hur kan vi illustrera dem?

$$c_n = |c_n| \cdot e^{i \arg(c_n)}$$

- * $|c_n|$ amplitudspektrum
- * $\arg(c_n)$ fasspektrum
- * $|c_n|^2$ effektspektrum

Exempel

Såghadsfunktioner

$$T = 2\pi, \quad \omega_0 = \frac{2\pi}{T} = 1$$

$$f(t) = t, \quad t \in [-\pi, \pi]$$

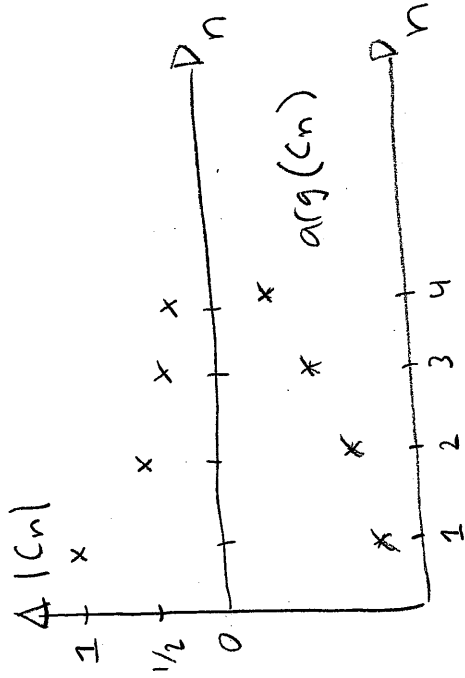
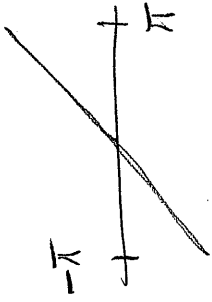
$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} t \cdot e^{-int} dt = \dots = \frac{j}{n} (-1)^n = \frac{j}{n} e^{i\pi/2} \cdot (e^{i\pi})^n$$

$$= \frac{1}{n} e^{i\pi(n+1/2)}$$

* Amplitudspektrum: $|C_n| = \frac{1}{n}$

* Fasespektrum: $\pi(n+1/2)$

* Effektspektrum: $|C_n|^2 = \frac{1}{n^2}$



Värmelednings ekvationen

Värmeledning i en dimensionell metallstav

Temperaturfördelning $T(x, t)$



Energiström: $J = -\lambda \frac{\partial T}{\partial x}$ hög $T \rightarrow$ låg T värmeströmmar



Energi balans i tunn skiva

$$c \rho A dx dT = A [J(x) - J(x+dx)] dt$$

\Rightarrow värmelednings ekvationen

$$\frac{\partial T}{\partial t} = -\frac{1}{c\rho} \frac{\partial J}{\partial x} = \frac{\lambda}{c\rho} \frac{\partial^2 T}{\partial x^2} = k \frac{\partial^2 T}{\partial x^2}$$

$$\boxed{\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}}$$

Variabelseparation

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

$$\text{Ansatz: } T(x, t) = X(x) \Theta(t) \longrightarrow$$

$$X'' = aX$$

$$\Theta' = ka\Theta$$

$$X_n(x) = A_n \sin\left(\frac{\pi n}{L} x\right)$$

$$\Theta_n = B_n e^{-k\left(\frac{\pi n}{L}\right)^2 t}$$

$$T(x, t) = \sum_n X_n(x) \Theta_n(t)$$

$$= \sum_n C_n \sin\left(\frac{\pi n x}{L}\right) e^{-k\left(\frac{\pi n}{L}\right)^2 t}$$

$$\text{Beyrandse villkor: } T(x, 0) = f(x) = \underbrace{\sum_n C_n \sin\left(\frac{\pi n x}{L}\right)}_{\text{Fouriersumme}} \Rightarrow C_n \text{ kan best\u00e4mmas}$$

mha fourierutveckling

Fouriertransformen

serie $t \in [-T/2, T/2]$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega_0 t}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-in\omega_0 t} dt$$

} $(T \rightarrow \infty) \rightarrow$

Integral $t \in [-\infty, \infty]$

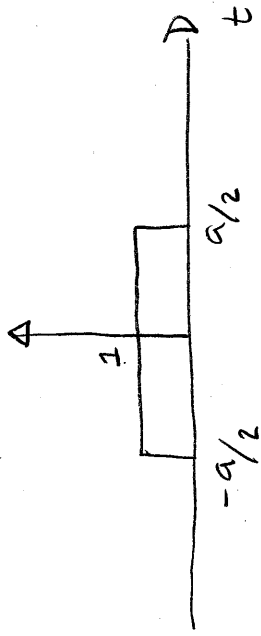
$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

• $F(\omega)$ är fouriertransformen av $f(t)$

• $f(t)$ är inversa fouriertransformen av $F(\omega)$

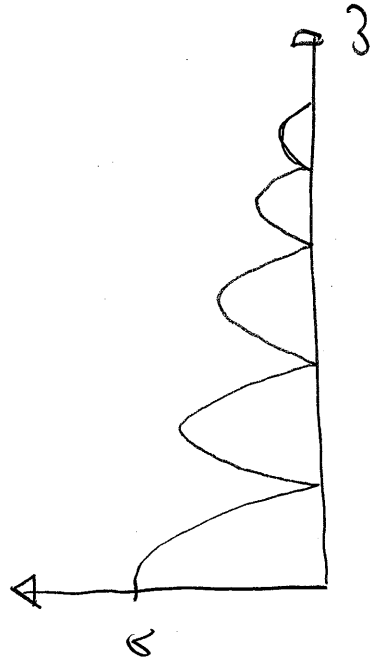
Exempel Blockfunktionen



$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = \int_{-a/2}^{a/2} e^{-i\omega t} dt$$

$$= \dots = \frac{2 \sin(\omega a/2)}{\omega}$$

• Amplitudenspektrum $|F(\omega)|$



innen diskret
nu kontinuerligt

Egenskaper för fouriertransformer

A, Linearitet

$$F(af+bg) = af(t) + bf(g)$$

C, Translation i ω

$$f(t)(\omega-a) = f(e^{iat} f(t))(\omega)$$

B, Translation i t

$$f(f(t-a)) = e^{-i\omega a} f(t)$$

D, skalning

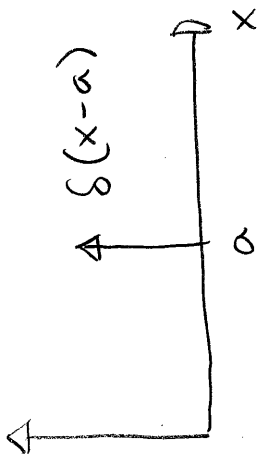
$$f(f(ct)) = \frac{1}{|c|} f(t)(\omega/c)$$

Vecka 3

Kap. 12 - 14

- * Diracs deltafunktion
- * Linsen som fouriertransformator
- * Konvolution

Diracs deltafunktion



Egenskap 1 $\int_{-\infty}^{\infty} \delta(x) dx = 1$

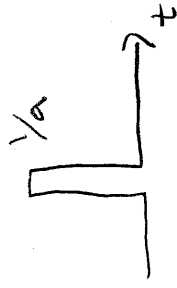
Egenskap 2 $\int_{-\infty}^{\infty} f(t) \delta(x) = f(0)$

Fouriertransformer

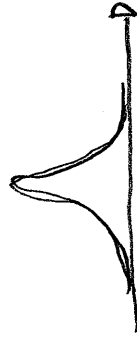
$$D(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-i\omega t} dt = 1$$

Olika definitioner

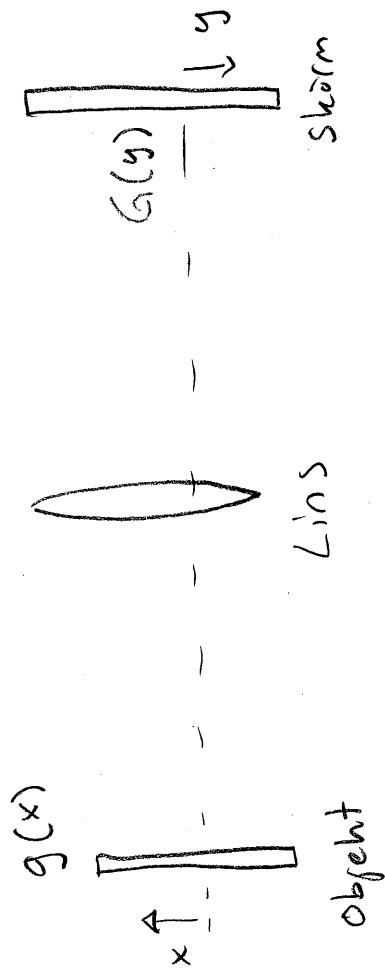
$$\delta(x) = \lim_{a \rightarrow 0} \begin{cases} 1/a & |t| \leq a/2 \\ 0 & |t| > a/2 \end{cases}$$



$$\delta(x) = \lim_{a \rightarrow 0} \frac{1}{\pi} \frac{a}{a^2 + t^2}$$



Linser som fouriertransformator



Amplituden på skärmen

$$G(y) = \int_{-\infty}^{\infty} g(x) \cdot e^{i 2\pi x \cdot y / (\lambda f)} dx$$

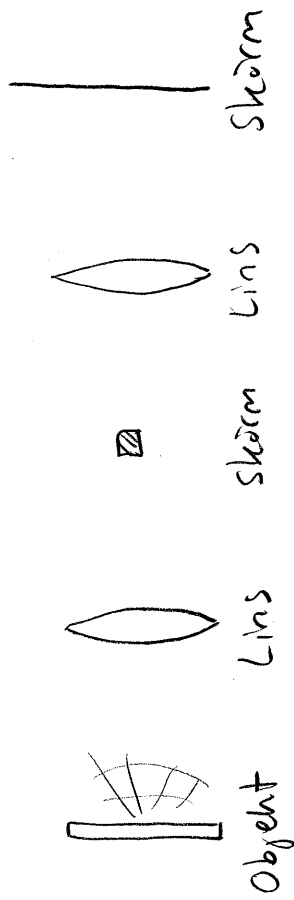
$$\left(\sim \sum_{\text{alle strålar}} \text{amplitud} \times e^{i \text{fas}} \right)$$

$$\text{infers } \omega = \left(\frac{2\pi}{\lambda f}\right) \cdot y \Rightarrow$$

$$G(\omega) = \int_{-\infty}^{\infty} g(x) e^{i \omega x} dx$$

- Ett föremål i objektplanet avbildas i fokallplanet som fouriertransformen av föremålet.

Frekvensfiltrering



Tillämpning: F-skontrastmikroskop

Konvolution

Exempel ofokuserad kamera



$f(t)$: fokuserad intensitet

$$\text{fokuserad intensitet: } h(t) = \int_{-\infty}^{\infty} f(t-\tau) g(\tau) d\tau$$

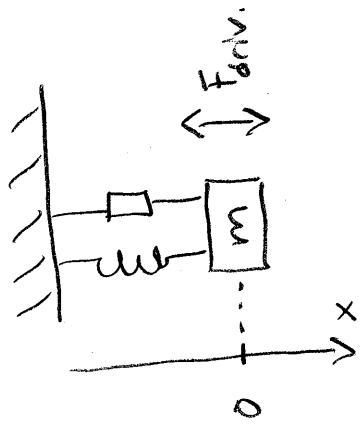
Fouriertransformer

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-i\omega t} dt = F(\omega) \cdot G(\omega)$$

Vecka 4

- * Diff. ekvationer
 - Rvikt i fräde med dämpning
 - Elektriska kretsar
 - Voltterras model
- * Sampling och diskret fouriertransform

Diff. ekv. I



$$m \frac{d^2 x}{dt^2} = -kx - \lambda \frac{dx}{dt} + F e^{i\Omega t} \quad (\text{diff. ekv})$$

m.a. fjäderkraft dämpning drivande kraft

Ansatz: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega) e^{i\omega t} d\omega \Rightarrow$

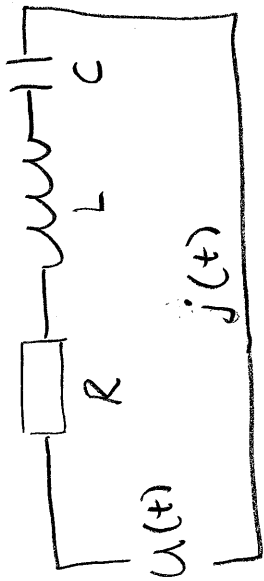
$$x(\omega) [\omega^2 - \omega_0^2 - 2i\beta\omega] = \left(-\frac{2\pi F}{m}\right) \delta(\omega - \Omega) \quad (\text{algebraisk ekvation})$$

Lös ut $x(\omega)$, transformera tillbaka \Rightarrow

$$x(t) = -\frac{F}{m} \frac{e^{i\Omega t}}{\Omega^2 - \omega_0^2 - 2i\beta\Omega} = c(\Omega) \cdot e^{i\Omega t}$$

- * Svänger med drivande kraftens frekvens.
- * Resonans för vissa Ω (amplituden $c(\Omega)$ blir stor)

Elektriske kretsar



$$u_R = R \cdot j, \quad u_L = L \frac{dj}{dt}, \quad u_C = \frac{q}{C}$$

$$\Rightarrow u(t) = R j(t) + L \frac{dj(t)}{dt} + \frac{q(t)}{C}$$

$$\text{Derivas} \Rightarrow \frac{du}{dt} = R \frac{dj}{dt} + L \frac{d^2j}{dt^2} + \frac{1}{C} j \quad (\text{diff. ekv.})$$

$$\text{Ansatz: } u(t) = U(\omega) e^{i\omega t}, \quad j(t) = J(\omega) e^{i\omega t}$$

$$\Rightarrow U(\omega) = \left(\underbrace{R}_{z_R} + \underbrace{i\omega L}_{z_L} + \underbrace{\frac{1}{i\omega C}}_{z_C} \right) J(\omega) \quad \left(\begin{array}{l} \text{algebraisk} \\ \text{ekv.} \end{array} \right)$$

* Z - impedans

Alla "gamla valige" kretsregler g\u00e4ller for impedanser

Kopplade första ordningens diff. ekv.

$$d^2y \frac{1}{dt^2} + \frac{dy}{dt} + y = 0$$

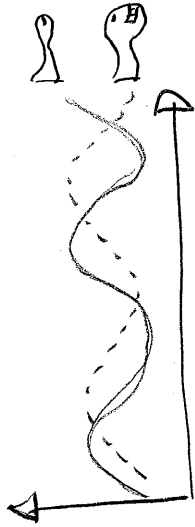
$$\text{in för } y_1 = y, \quad y_2 = \frac{dy}{dt}$$

$$\Rightarrow \left\{ \begin{array}{l} \frac{dy_2}{dt} + y_2 + y_1 = 0 \\ \frac{dy_1}{dt} = y_2 \end{array} \right.$$

* En n:te ordningens diff. ekv kan
gävas om till n st kopplade första
ordningens diff. ekv.

Kopplede diff. ekv. - Volterres model

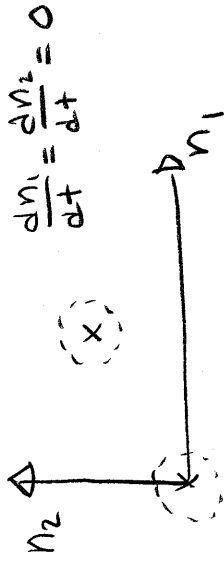
observation



Modell

$$\begin{cases} \frac{dn_1}{dt} = a n_1 - b n_1 n_2 \\ \frac{dn_2}{dt} = -c n_2 + d n_1 n_2 \end{cases}$$

Lösning kring jämviktspunkter

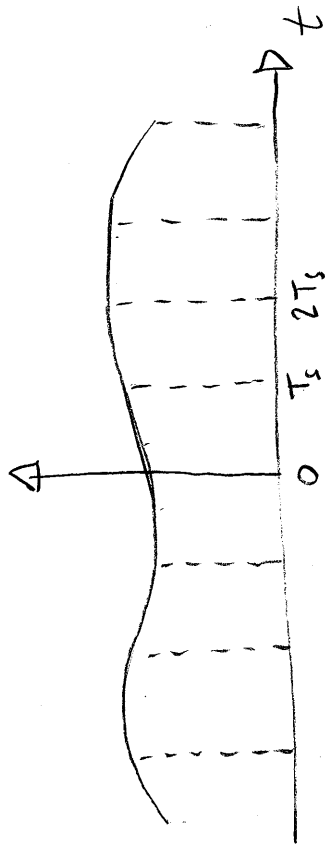


Linearisering + ansatz $\vec{z} = e^{\lambda t} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \Rightarrow$

vi fann: $n_1^* = n_2^* = 0$ instabil jämviktspunkt \curvearrowright
 $n_1^* = \frac{c}{d}, n_2^* = \frac{a}{b}$ stabil jämviktspunkt \curvearrowright

Harmoniska (sin, cos) lösningar

Sampling



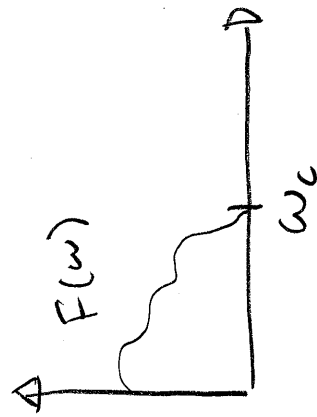
$$T_s = \text{Samplingperiod}, \quad \omega_s = \frac{2\pi}{T_s}$$

} samplings frekvens

Diskret signal:

$$f[n] = f(nT_s), \quad n = 0, \pm 1, \pm 2, \dots$$

- Nyquistteoremet



Vi måste sample med mer än dubbla ω_c
om signalen ska kunna rekonstrueras
fullständigt.

annars aliasing (spök frekvens)

Discret Fouriertransform

$$f(t) \rightarrow f[n]$$

$$\int \rightarrow \sum$$

$$f[n] = \frac{1}{N} \sum_{k=0}^{N-1} F[k] e^{i2\pi nk/N}$$

$$F[k] = \sum_{n=0}^{N-1} f[n] e^{-i2\pi nk/N}$$

}

* Dator - FFT

Laplace transform

reell

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt, \quad s = \sigma + i\omega, \quad \sigma > 0$$

- Signaler som slås på vid $t=0$
- Konvergens bättnen är fouriertransformer

- Derivat

$$f'(t) = \frac{df}{dt} \quad \xrightarrow{\mathcal{L}} \quad F'(s) = F(s) \cdot s - f(0)$$

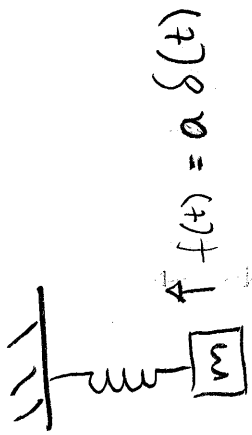
- Integral

$$g(t) = \int_0^t f(\tau) d\tau \quad \xrightarrow{\mathcal{L}} \quad G(s) = \frac{1}{s} F(s) - g(0) \frac{1}{s}$$

Vecka 5

- * Laplace transform
- * Överfäringsfunktion
- * Partielle diff. ekv.
- * Åtströppade system

Driven oscillator med Laplace

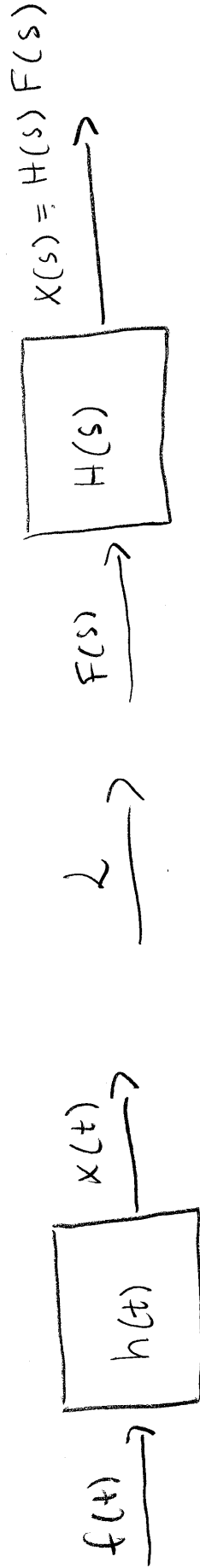


$$m \frac{d^2 x}{dt^2} = -kx + a \delta(t)$$

Laplace transform \Rightarrow

$$\underbrace{X(s)}_{\text{utsignal}} = \underbrace{H(s)}_{\text{överföringsfunktion}} \cdot \underbrace{F(s)}_{\text{insignal}}$$

Generalisering: För alla linjära system



* $h(t)$ är impulssvar, för $\delta(t)$ insignal $\Rightarrow X(t) = h(t)$

Översättningsfunktioner

Generellt kan vi partialbröksuppdele $H(s) \Rightarrow$

$$H(s) = K \cdot \sum_{k=1}^N R(p_k) \frac{1}{s - p_k}$$

$$h(t) = K \sum_{k=1}^N R(p_k) e^{p_k \cdot t}$$

En eller flera $\operatorname{Re}(p_k) > 0$: Instabilt system

Alla $\operatorname{Re}(p_k) < 0$: Stabilt system

Frekvenssvar

Harmonisk insignal: $f(t) = f_0 e^{i\omega t}$, stabilt system

\Rightarrow för $t \rightarrow \infty$ (starte i insvagningsförlopp)

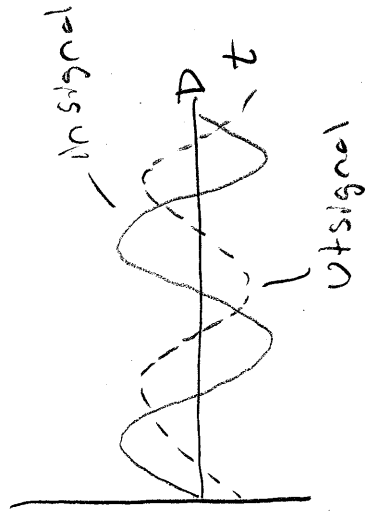
$$X(t) = H(i\omega) f(t) = H(i\omega) f_0 e^{i\omega t}$$

Kvot mellan in och utsignal

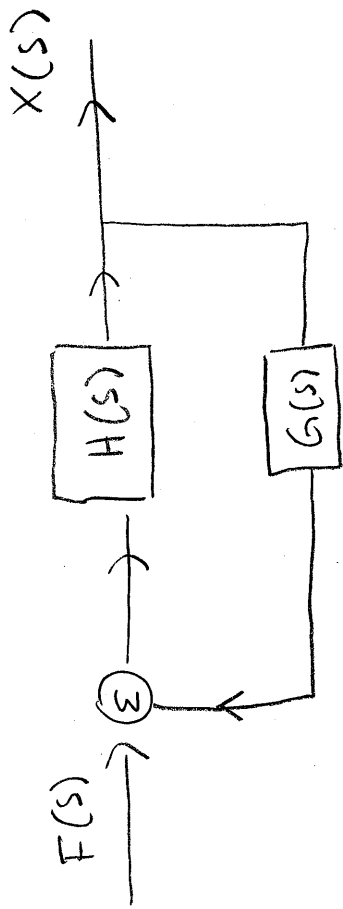
$$\frac{X(t)}{f(t)} = H(i\omega) = |H(i\omega)| e^{i \arg(H(i\omega))}$$

Amplitud
förstärkning

Fasförskjutning



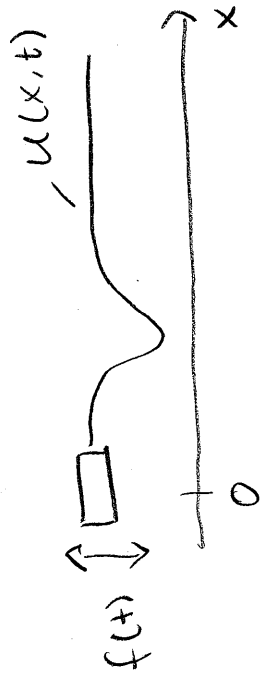
Återkopplade system



$$X(s) = \frac{H(s)}{1 - H(s)G(s)} F(s) = \bar{H}(s) F(s)$$

- o Regler insignalen m.h.a. information om utsignalen
- Kan stabilisera instabilt system

Laplace och partiella diff. ekv.



Pishnart problemet:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0, \quad x > 0, t > 0$$

$$\mathcal{L} \left[\frac{\partial^2 U(x,s)}{\partial x^2} \right] = \frac{\delta^2}{c^2} U(x,s)$$

Allmän lösning: $U(x,s) = a(s) e^{sx/c} + b(s) e^{-sx/c}$

$$\mathcal{L}^{-1} \rightarrow u(x,t) = f(t - x/c), \quad \text{vågor} \rightarrow$$

Vecka 6

- * Repetition av vektoranalys
- * Maxwells ekv.

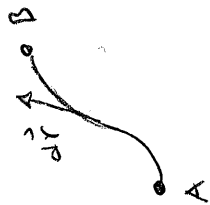
Vektoranalys

- Vektor i 3D $\vec{a} = (a_x, a_y, a_z)$, $\vec{b} = (b_x, b_y, b_z)$
- Skalarprodukt $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = a_x b_x + a_y b_y + a_z b_z$
- Vektorprodukt $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$
- Skalarfält $\phi(x, y, z)$
- Vektorfält $\vec{F}(x, y, z)$
- Nabla operatören $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$
- Nabla på skalarfält $\nabla \phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \right)$
- Nabla skalar på vektorfält $\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$
- Nabla vektoriellt på vektorfält $\nabla \times \vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$

Vektoranalys - integraler

* Linjeintegral

$$\int_{\Gamma_{AB}} \vec{a} \cdot d\vec{r}$$



sluten kurva



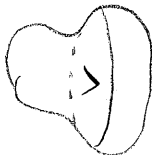
$$\oint_{\Gamma(S)} \vec{a} \cdot d\vec{r}$$

* Ytintegral

$$\int_S \vec{a} \cdot d\vec{s}$$



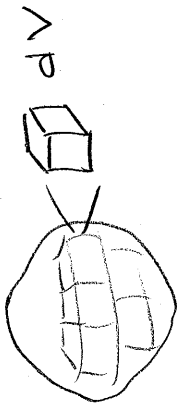
sluten yte



$$\int_{S(V)} \vec{a} \cdot d\vec{s}$$

* Volumintegral

$$\int \phi \, dV$$



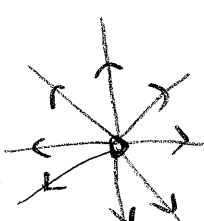
Integralsetser:

$$\oint_{\Gamma(S)} \vec{a} \cdot d\vec{r} = \int_S (\nabla \times \vec{a}) \cdot d\vec{s} \quad \text{Stokes setser}$$

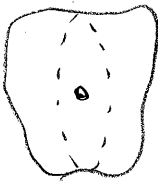
$$\int_{S(V)} \vec{a} \cdot d\vec{s} = \int_V \nabla \cdot \vec{a} \, dV \quad \text{Gauss setser}$$

Maxwells ekv

A) Coulombs lag


$$\vec{E} = \vec{e}_r \frac{q}{4\pi \epsilon_0 r^2}$$

Lägg en volym runt \Rightarrow


$$\int_{S(V)} \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

Gäller för godtyckliga volymer som innehåller q , ... \Rightarrow

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

B) Magnetiske ledningar finns ej

Magnetiske fältlinjer bildar slutna kurvor

$$\int_{S(V)} \vec{B} \cdot d\vec{s} = 0 \quad (\text{Gauss}) \quad \Rightarrow$$

$$\int_V \nabla \cdot \vec{B} \, dV = 0$$

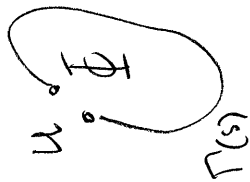
Godtycklig volym \Rightarrow

$$\nabla \cdot \vec{B} = 0$$

Maxwells ekr.

C | Faradays induktionslag

$$u = - \frac{d\Phi}{dt}$$



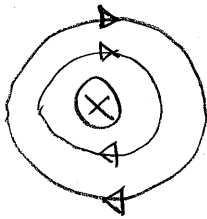
$$\Phi = \int_S \vec{B} \cdot d\vec{s}$$

$$u = \oint_{\Gamma(s)} \vec{E} \cdot d\vec{r}$$

+ Stokes sets ... \Rightarrow

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

D | Amperes lag



$$B = \mu_0 \frac{I}{2\pi r}$$

$$\Rightarrow \oint_{\Gamma(s)} \vec{B} \cdot d\vec{r} = \mu_0 \int_S \vec{j} \cdot d\vec{s}$$

Stokes sets, godtycklis yta ... \Rightarrow

$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

Denne fel, kontinuitets ekv. \Rightarrow

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

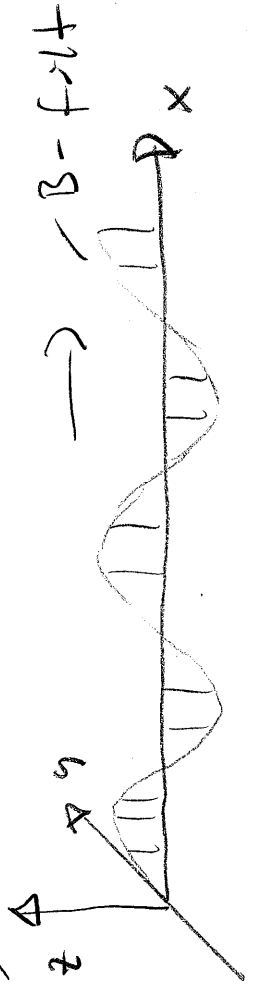
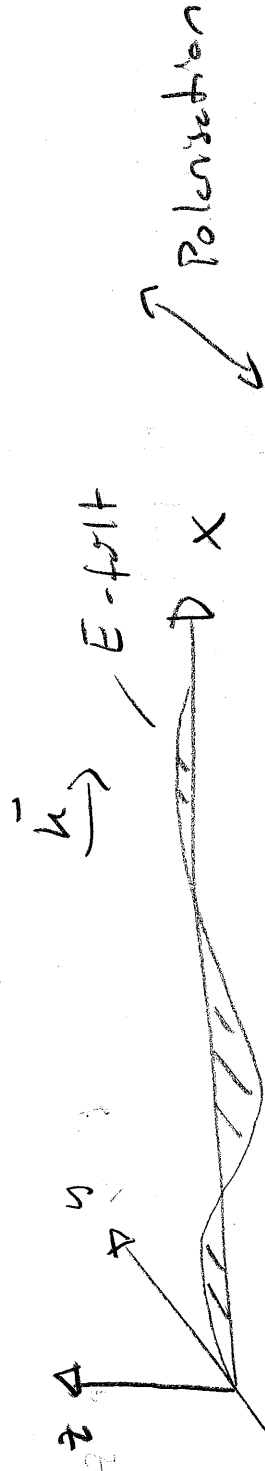
Maxwells ekv. i Vakuum ($\rho=0, \vec{j}=0$)

Ansetz: $\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$, $\vec{B} = \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$

$$\left\{ \begin{array}{l} \nabla \cdot \vec{E} = 0 \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \end{array} \right.$$

$$\left\{ \begin{array}{l} \vec{k} \cdot \vec{E} = 0 \\ \vec{k} \cdot \vec{B} = 0 \\ \vec{k} \times \vec{E} = \omega \vec{B} \\ \vec{k} \times \vec{B} = -\mu_0 \epsilon_0 \omega \vec{E} \end{array} \right.$$

$\Rightarrow \vec{k}$ i x-led, \vec{E} i y-led $\Rightarrow \vec{B}$ i z-led



Vedde 7

* Maxwell's ekv. i "snöle" medver

Snälla Material (isolatorer)

\vec{E} -fält \rightarrow polarisering av bundna laddningar

$\rho = \rho_{fri} + \rho_{bund}$

* $\rho_{bund} = -\nabla \cdot \vec{P}$

* Polarisation $\vec{P} = \epsilon_0 \chi_e \vec{E}$

\Rightarrow Modified ME I

$\nabla \cdot \vec{E} = \rho / \epsilon$

\vec{B} -fält \rightarrow magnetisering av bundna strömmar

$\vec{j} = \vec{j}_{fri} + \vec{j}_{bund}$

* $\vec{j}_{bund} = \nabla \times \vec{M} + \frac{d\vec{P}}{dt}$

* Magnetisering $\vec{M} = \chi_m \vec{H}$

\Rightarrow Modified ME VI

$\nabla \times \vec{B} = \mu \vec{j}_{fri} + \epsilon \mu \frac{\partial \vec{E}}{\partial t}$

Samma ekv. som tidigare men

$$\epsilon = \epsilon_0 \epsilon_r = \epsilon_0 (1 + \chi_e)$$

$$\mu = \mu_0 \mu_r = \mu_0 (1 + \chi_m)$$

Snälle metaller

Ljshastigheten i ett metall:

$$c = \frac{1}{\sqrt{\epsilon_r \mu_r}} = \frac{1}{\sqrt{\epsilon_r \mu_r}} \cdot c_{\text{vakuum}}$$

$\epsilon_r \geq 1$, $\mu_r \geq 1 \Rightarrow$ lägre ljusfart i metall