Lund Observatory
Department of Astronomy and Theoretical Physics
LUND

Exercises Fundamental Astronomy, chapters 10, 11 and 14

1. One can show that stars cannot have masses much smaller than 0.1 $M_\odot$ or larger than 100 $M_\odot$.

a) Which are the reasons for this?

b) When discussing stellar structure and evolution, one often divides stars into massive and less massive stars. What is approximately the dividing mass limit? How is the structure of the two types differing during main sequence evolution?

c) Helium burning starts under drastically different conditions in massive and less massive stars. Which is the main difference, and what consquences will it have for the reaction rates?

2. For en star in equilibrium we have:

$$\frac{dP(r)}{dr} = -\frac{GM(r)}{r^2}\rho.$$

a) Show that

$$\frac{d}{dr}\left(P(r) + \frac{GM(r)^2}{8\pi r^4}\right) < 0$$

where $M(r) = \int_0^r 4\pi r^2 \rho dr$. Derive a lower limit for the central pressure, P_c, as a function of the total mass of the star, $M_\star$, and its radius, $R_\star$.

b) Suppose that the density of a star is given by:

$$\rho(r) = \rho_c \left(1 - \frac{r}{R_\star}\right)$$

where ρ_c is the central density. Derive an expression for the total mass of the star and calculate the ratio between the central density and the mean density, $(\rho_c/\bar{\rho})$.
Derive also an expression for the pressure as a function of radius, and show from this that rhe central density is given by $P_c = \frac{5}{4\pi}\frac{GM_\star^2}{R_\star^4}$. (One can assume that $P(R_\star) = 0$).

3. The structure of a massive star in a stadium where the energy production almost has ceased can be shown by a schematic figure. Draw such a figure and specify the main elements found in different regions of the star.

4. ^{4}He is produced both at Big Bang and in stars. While the production of helium only took a few minutes in the early Universe, it can last for billions of years in a star like the Sun. What is the main reason for this?

5. By approximating the Gamow peak with a gaussian one can show that the power produced per mass unit can be written as

$$\epsilon_{xy} = \epsilon_0 \rho X_x X_y T^{-2/3} e^{-\beta T^{-1/3}}$$

where X_x and X_y are the mass fractions of the particles x and y respectively, while ϵ_0 and β are constants.
For the pp-chain we have $\epsilon_0 \approx 2.4 \cdot 10^3$ and $\beta \approx 3381$. For the CNO cycle the corresponding constants are $\epsilon_0 \approx 7.2 \cdot 10^2 4$ and $\beta \approx 15230$
Assume that within a limited temperature interval the following approximation is valid

$$\epsilon_{xy} \approx \epsilon_1 \rho X_x X_y T^{\nu}$$

Determine the value of ν for the pp-chain at a temperature $T \approx 1.5 \cdot 10^7$ K and for the CNO cycle at $T \approx 2.0 \cdot 10^7$ K.

6. Elements with mass greater than $A \approx 60$ can be synthesized in different ways. Describe in which kinds of astronomical objects the different processes take place.

7. Supernovae of type II are caused by a collapse of a stellar core from white dwarf densities ($\approx 10^9$ kg m^{-3}) to neutron star densities ($\approx 10^{18}$ kg m^{-3}). Estimate the amount of energy released at a supernova event of type II. One can assume that the potential energy of a body of mass M and radius R is

$$\frac{G \cdot M^2}{R}$$

A type II supernova has, during the first three to four months, a luminosity od about $10^8 L_\odot$. During the supernova explosion the outer layers of the star (around 10 solar masses) are expelled into space with speeds of about 10^4 km/s. Show that most of the energy released emerges in a form not readily detectable. Give a suggestion as to what this form might be.
($1\ M_\odot = 2 \cdot 10^{30}$ kg, $1\ L_\odot = 3.85 \cdot 10^{26}$ Js^{-1})