Hand in problems 1, Nuclear Structure Theory 2009

1.1. Assume the semiempirical mass formula in the form

$$m(N,Z) = NM_n + ZM_H - \frac{1}{c^2}B(N,Z)$$

with

$$B(N,Z) = a_v A\left(1 - \kappa_v\left(\frac{(N-Z)}{A}\right)^2\right) - a_s A^{2/3}\left(1 - \kappa_s\left(\frac{(N-Z)}{A}\right)^2\right) - a_c\frac{Z^2}{A^{1/3}}$$

where

$$a_v = 15.494\,\text{MeV}\ \ a_s = 17.944\,\text{MeV}\ \ a_c = 0.705\,\text{MeV}\ \ \kappa_v = \kappa_s = 1.7826$$

(somewhat simplified Myers-Swiatecki parameters)

a) Find an expression for the β stability line in the form $N - Z = f(A)$. Note that the binding energy can alternatively be written:

$$B(N,Z) = a_v A - a_s A^{2/3} - a_c\frac{Z^2}{A^{1/3}} - a_{sym1}\frac{(N-Z)^2}{2A} + a_{sym2}\frac{(N-Z)^2}{2A^{4/3}}$$

Even though this equation is appropriate only for limited values of A, it is interesting to consider the limit when $A \to \infty$. Do this!

Hint: The The line of beta-stability is defined from:

$$\left(\frac{\partial m}{\partial N}\right)_{A=const} = 0$$

This derivative is straightforward to calculate if the mass is expressed as a function of N and A, i.e. Z is replaced by $(A - N)$ in the mass formula. Now, the requirement that the derivative is zero corresponds to a relation $g(N,A) = 0$. We are asked to derive $(N - Z)$ as a function of A. This expression is easy to find if the relation $g(N,A) = 0$ is rewritten so that $(N - Z)$ and A ($N = (1/2)(N - Z) + (1/2)A$) are used as independent variables instead.

b) Calculate numerical values according to the formulas derived for $A = 100$ [and $A = 200, 300$]. The masses of the hydrogen atom and the neutron, respectively are: $M_H c^2 = 938.77$ MeV, $M_n c^2 = 939.55$ MeV. Compare with Fig. 3.7 in the text book.

1.2. A proton in a spherical subshell will see a different average potential than a neutron due to the Coulomb interaction. Consider now a proton in a $1h_{11/2}$ ($N = 5$) subshell. Assume that the wave-function is of harmonic oscillator type,

$$\phi(n\ell sj\Omega) = R_{n\ell}(r) \sum_{\Lambda\Sigma} \langle \ell s\, \Lambda\Sigma | \ell sj\Omega \rangle\, Y_{\ell\Lambda} \chi_{s\Sigma}$$

Give an explicit expression for this wave-function including the normalization constant (you need not give explicit expressions for the spherical harmonics).

b) Assume that the Coulomb potential is $V_C(r)$. Set up the first order perturbation expression of the energy shift if the Coulomb potential is given by $V_C(r)$. You are supposed to solve angular but not radial integrals, i.e. your answer will contain an integral in the r-coordinate.

c) Now assume that the proton is moving in a Coulomb potential of a ^{114}Sn ($Z = 50, N = 64$) core. Use the expression for the Coulomb potential derived in exercise 5.3 (exercise 6.11 in the text book):

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{r} \qquad r \geq R$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{R}\left(-\frac{1}{2}\left(\frac{r}{R}\right)^2 + \frac{3}{2}\right) \;;\; (r \leq R)$$

(with $R = r_0 A^{1/3}$, $r_0 = 1.2$ fm)

Furthermore, make the drastic approximation that the full charge of the proton is at distance from the center corresponding to the $\langle r^2 \rangle$-value of the $1h_{11/2}$ wave-function, i.e. it is concentrated in a shell with the radius $\sqrt{\langle r^2 \rangle}$. Determine a numerical value for the resulting energy shift. Choose reasonable values for the oscillator frequency $\hbar\omega$, from which you can calculate for example the 'length constant' $(\frac{M\omega}{\hbar}) = (\frac{Mr_0^2}{\hbar^2}\frac{1}{r_0^2}\hbar\omega)$ (see p. 63). Which value of the energy shift would you get if the proton was placed at the $\langle r^2 \rangle$-value of a $2d_{5/2}$ proton instead. Do these values appear reasonable when comparing with the depth of the single-proton potential?