THERMODYNAMICS
AND
STATISTICAL PHYSICS

Answers to problems

Week 1

1.

$$\Delta S(\text{water}) = 1.31 \text{ kJ}/^\circ\text{K}$$

$$\Delta S(\text{reservoir}) = -1.12 \text{ kJ}/^\circ\text{K}$$

$$\Delta S(\text{water} + \text{reservoir}) = 0.19 \text{ kJ}/^\circ\text{K}$$

2.

$$C_p - C_v = \left(\frac{\partial E}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$$

3. a)

$$T_2 = 2^{-2/3} T_1$$

b)

$$T_2 = T_1$$

c)

$$\Delta S = 0 \quad \text{and} \quad \Delta S = Nk \ln 2 \quad \text{respectively}$$

4. a)

$$\Delta S = \int_{300}^{400} \frac{C_p(T)}{T} \mathrm{d}T = 10.99 \text{ J}/(\text{mole } ^\circ\text{K})$$

b)

$$\Delta S = \int_{300}^{400} \frac{C_V(T)}{T} \mathrm{d}T = 8.60 \text{ J}/(\text{mole } ^\circ\text{K}) \qquad \text{där} \quad C_V = C_p - R$$

5.

$$\gamma = \frac{4\pi^2 m V_0}{\tau^2 A(Ap_0 + mg)}$$

6. It can be shown that $C_V = C_V(T)$ which implies that both a) and b) are possible.

7. —

Week 2

8.

$$0 < x < L; \quad -\sqrt{2mE} - \frac{m}{\sqrt{2mE}}\,\delta E < p_x < -\sqrt{2mE}$$

and

$$0 < x < L; \quad \sqrt{2mE} < p_x < \sqrt{2mE} + \frac{m}{\sqrt{2mE}}\,\delta E$$

9. a)

$$P(x) = \frac{1}{\pi\sqrt{A^2 - x^2}}$$

$$v = -\omega\sqrt{A^2 - x^2} \qquad \Longrightarrow \qquad P \sim \frac{1}{v}$$

b) Same as in a)

10. a) —

b)

$$\overline{p} = \frac{2\overline{E}}{3V}, \quad \text{where} \quad \overline{E} = \frac{\hbar^2\pi^2}{2m} \cdot 3\frac{\overline{n_z^2}}{L_z^2}$$

11.

$$\Omega(E) = \frac{\pi}{4}\left(\frac{2mL^2}{\hbar^2\pi^2}\right)^{3/2}\sqrt{E}\;\delta E$$

12. a)

$$\Omega(E) = \frac{(M+N-1)!}{M!(N-1)!}$$

b)

$$P = \frac{M(M-1)\cdots(M-k+1)(N-1)}{(M+N-1)(M+N-2)\cdots(M+N-k-1)} \approx \frac{M^k N}{(M+N)^{k+1}}$$

13.

$$\frac{1}{T} = \frac{k}{h\nu}\ln\left(\frac{E/N + h\nu/2}{E/N - h\nu/2}\right)$$

$$\frac{E}{Nh\nu} = \frac{1}{2} + \frac{1}{e^{h\nu/(kT)} - 1}$$

14. —

15. —

Week 3

16. —

17.

$$S = -k \sum_r P_r \ln P_r$$

18. —

19. a)

$$P_1/P_0 = e^{-\hbar\omega/(kT)}$$

b)

$$\overline{E} = \frac{\hbar\omega}{2} \cdot \frac{1 + 3e^{-\hbar\omega/(kT)}}{1 + e^{-\hbar\omega/(kT)}}$$

20.

$$N_+ - N_- = N_0 \frac{\mu H}{kT} \qquad \Longrightarrow \qquad \text{absorbed effect} \sim \frac{1}{T}$$

21. a)

$$\overline{E} = \frac{2\varepsilon N_A}{2 + e^{\varepsilon/(kT)}}$$

b)

$$S = kN_A \ln(1 + 2e^{-\varepsilon/(kT)}) + \frac{2\varepsilon N_A}{T(2 + e^{\varepsilon/(kT)})}$$

22. a)

$$\overline{E_k} = \frac{3}{2}kT$$

b)

$$\overline{E_p} = kT + mgL\left(1 - e^{mgL/(kT)}\right)^{-1}$$

23. a)

$$p = \frac{R}{V}(\nu_1 T_1 + \nu_2 T_2) \quad \text{where} \quad V = V_1 + V_2 \quad \text{with} \quad V_i = \frac{\nu_i R T_i}{p_i}$$

b)

$$\Delta S_{\text{different}} = R\nu_1 \ln\left[\frac{V}{V_1}\left(\frac{T}{T_1}\right)^{3/2}\right] + R\nu_2 \ln\left[\frac{V}{V_2}\left(\frac{T}{T_2}\right)^{3/2}\right]$$

c)

$$\Delta S_{\text{identical}} = \Delta S_{\text{different}} - R\nu_1 \ln\left(\frac{\nu_1 + \nu_2}{\nu_1}\right) - R\nu_2 \ln\left(\frac{\nu_1 + \nu_2}{\nu_2}\right)$$

24.

$$\bar{l} = Na \tanh\left(\frac{mga}{kT}\right)$$

25.

$$C_S = \left(\frac{\partial \overline{E}}{\partial T}\right)_S = N_A k = R$$

Week 4

26. a)

$$\overline{x} = \frac{Mg}{\alpha}$$

b)

$$\overline{(x - \overline{x})^2} = \frac{kT}{\alpha}$$

c)

$$M = \frac{\sqrt{\alpha kT}}{g}$$

27. a)

$$L = Na$$

b)

$$\delta L = N\delta \ \tanh\left(\frac{mg\delta}{kT}\right)$$

28.

$$T = \frac{m\omega^2\gamma d}{k}$$

29. a)

$$\text{Monoatomic gas}: \ c_v = \frac{3}{2}R$$
$$\text{Twoatomic gas}: \ c_v = \frac{5}{2}R$$

b)

$$\text{Monoatomic gas}: \ c_v = 2.98 \ \text{cal}/(\text{mole}\ ^\circ\text{K})$$
$$\text{Twoatomic gas}: \ c_v = 4.97 \ \text{cal}/(\text{mole}\ ^\circ\text{K})$$

c) —

30. a)

$$E = J(2n - N + 1)$$

b)

$$S = k(\ln(N-1)! - \ln n! - \ln(N-1-n)! + \ln 2)$$

c)

$$n = \frac{N-1}{e^{2J/(kT)} + 1}$$

31.

$$\overline{E} = \frac{h\nu}{e^{h\nu/(kT)} - 1}$$

$$C_v = \frac{h^2\nu^2}{kT^2}\frac{e^{h\nu/(kT)}}{\left(e^{h\nu/(kT)} - 1\right)^2}$$

$$C_v \longrightarrow 0 \quad \text{when} \quad T \longrightarrow 0$$

32.

$$C_v = \frac{\hbar^2\omega^2}{kT^2}\frac{Ne^{\hbar\omega/(kT)}}{\left(e^{\hbar\omega/(kT)} - 1\right)^2}$$

$$C_v \longrightarrow 0 \quad \text{when} \quad T \longrightarrow 0$$

$$C_v \longrightarrow Nk \quad \text{when} \quad T \longrightarrow \infty$$

33.

$$C_V = \frac{5}{2}Nk - Nk\beta^2\frac{\left(m\omega^2R^2/2\right)^2 e^{-\beta m\omega^2R^2/2}}{\left(1 - e^{-\beta m\omega^2R^2/2}\right)^2}$$

$$C_V \longrightarrow \frac{5}{2}Nk \quad \text{when} \quad R \longrightarrow \infty$$

34.

$$Z = \frac{2\mathcal{J}}{\hbar^2} \cdot kT$$

35. a)

$$\frac{P(I=0)}{P(I=1)} = \frac{1}{3} \cdot e^{\hbar^2/(kT\mathcal{J})}$$

b)

$$c_v(\text{rotation}) = k$$

Week 5

36. a)

$$S = -k\sum_r \left[\overline{n}_r \ln \overline{n}_r + (1 - \overline{n}_r)\ln(1 - \overline{n}_r)\right]$$

b)

$$S = -k\sum_r \left[\overline{n}_r \ln \overline{n}_r - (1 + \overline{n}_r)\ln(1 + \overline{n}_r)\right]$$

c)

$$S = -k\sum_r \overline{n}_r \ln \overline{n}_r$$

37.

$$S = \frac{N\pi^2 k^2 T}{2\mu_0} + \dots$$

38. a)

$$\mu/\varepsilon = 1.6614 \qquad \overline{n}_r(0) = 1.6809$$

b)

$$\mu/\varepsilon = -0.3216 \qquad \overline{n}_r(0) = 2.6363$$

39.

$$pV = \frac{2}{3}E$$

40.

$$pV = NkT(1 \pm \frac{N\lambda^3}{8\sqrt{2}\,V}) \quad \text{där} \quad \lambda = \frac{h}{\sqrt{2\pi mkT}}$$

where + holds for fermions and − för bosons

41.

$$T_f = \frac{1}{2}T_i$$

42. a)

$$dS = \frac{4}{3T}\overline{u}dV + \frac{V}{T}\left(\frac{\partial \overline{u}}{\partial T}\right)dT$$

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{V}{T}\left(\frac{\partial \overline{u}}{\partial T}\right)$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \frac{4\overline{u}}{3T}$$

b) —

43. a)

$$P = 1.78\ 10^{15}\ \ \mathrm{W}$$

b)

$$P/A = 1.42\ 10^{8}\ \ \mathrm{W/m^2}$$

c)

$$\lambda = 48\ 10^{-10}\ \ \mathrm{m}$$

44. a)

$$T = T_0\sqrt{\frac{R}{2L}}$$

b)

$$T = 266\ {}^\circ\mathrm{K}$$

45.

$$T = 44.2\ ^\circ\mathrm{K}$$

Week 6

46. —

47. a)

$$a = \frac{9}{8} R V_k T_k \qquad b = \frac{V_k}{3}$$

b)

$$p_k = \frac{3RT_k}{8V_k}$$

c)

$$\left(p' + \frac{3}{(V')^2}\right)\left(V' - \frac{1}{3}\right) = \frac{8T'}{3}$$

48. a)

$$T = 195.2\ ^\circ\mathrm{K}$$

b)

$$\begin{array}{llll} \text{solid} & \longrightarrow & \text{gas} & L_{sg} = 31211\ \mathrm{J/mole} \\ \text{liquid} & \longrightarrow & \text{gas} & L_{lg} = 25466\ \mathrm{J/mole} \end{array}$$

c)

$$L_{sl} = 5745 \mathrm{J/mole}$$

49.

$$\mu = \mu(p, T, z) = -kT \left\{ \ln \frac{kT}{p} + \frac{3}{2} \ln \left(\frac{2\pi m k T}{h^2} \right) - \frac{mgz}{kT} \right\}$$

$$\mu = \text{constant} \quad \Longrightarrow \quad p = p_0 \left(\frac{T}{T_0} \right)^{5/2} \cdot e^{(1/T - 1/T_0)\mu/k} \cdot e^{-(z/T - z_0/T_0)mg/k}$$

where p_0, T_0, z_0 is a reference point. If, in addition, the temperature is constant, i.e. $T = T_0$, and $z_0 = 0$ one gets

$$p = p_0 e^{-mgz/(kT)}$$

which is the "isothermic barometer formula".

50.

$$C_V = \frac{12}{5} \pi^4 k N \left(\frac{T}{\Theta_D} \right)^3$$