Thermodynamics and Statistical Physics (FMF150)

Theory questions for the exam, January 8, 2018

1. Describe the difference between an adiabatic and an isothermal process. Then, derive for each of the two processes an expression $P = P(V)$ for how the pressure of an ideal gas depends on the volume. Assume quasi-static conditions.

2. Give a definition of a photon gas and write down the appropriate expression for the partition function Z. Then, use Z to derive an expression for the distribution function, the so called Planck distribution.

3. Give a short description of Gibbs paradox. Then, choose a simple example to illuminate why the paradox appears and how it can be explained. You may use the relation

$$S = Nk \left[\ln \left(V \left(\frac{4\pi mU}{3Nh^2} \right)^{3/2} \right) + \frac{3}{2} \right]$$

   without proof.

4. Describe in words how the slope of the phase equilibrium line in a (P, T)-diagram is related to the changes in entropy and volume which occur when the line is crossed. Then, use the relation to show how the latent heat of transformation is related to the slope of the phase equilibrium line.

5. Describe, using your own words and a schematic figure, the Debye approximation for describing lattice vibrations. Your description should contain a good motivation for the approximation. It should also contain a definition of the Debye temperature and the Debye frequency as well as an explanation of what a phonon is.

6. Derive an expression for the entropy of an ideal gas, assuming that the gas can be treated classically.

7. Formulate the four thermodynamic laws (including the zeroth law).

8. A table of molar heat capacities c_V (in cal mol^{-1} K^{-1}) contains among others the following entries

He	2.98
N_2	4.95
CO_2	6.77

   Explain why $c_V(He) < c_V(N_2) < c_V(CO_2)$.

9. a) What is the difference between an extensive and an intensive variable? Give two examples of variables of each kind.
   b) What is the difference between an equilibrium state and a stationary state?

10. Define the partition function and show how the mean energy and entropy can be calculated from the partition function.

11. Describe how c_V for a metal depends on T at very low temperatures. What is the physical explanation to this dependence?

12. Give a description of
    - an equilibrium state
    - a quasi-static process

13. What is
    - a statistical ensemble
    - a canonical ensemble
    - a grand canonical ensemble
    - a microcanonical ensemble

14. What is
    - the triple point
    - the critical point
    - the vapor pressure
    Use a (P,T)-diagram to illustrate the answer.

15. Describe, e.g. by using schematic figures, the behavior of the three distribution functions (Fermi-Dirac, Bose-Einstein and Maxwell-Boltzmann) in the limits $T \to \infty$ and $T \to 0$. How can the differences and similarities be explained in physical terms?

16. Describe van der Waals equation of state. Illustrate it by drawing schematic isotherms for different temperatures in a (P,V)-diagram.

17. Using the partition function, derive the equipartition theorem. The equipartition theorem is generally not applicable at low temperatures, which physical relation (or principle) is the reason for its break down?

18. Show, or argue for, that $dS > Q/T$ for a *non* quasi-static process. Also, discuss the notion of reversible and irreversible processes and, in particular, if there is any connection to the processes being quasi-static or not.

19. Show, or argue for, that the chemical potential $\mu = 0$ when the number of particles (in an isolated system) is not conserved. Also, based on the expression for dG, show the relation between μ and G for a system of N particles at temperature T and pressure P.

20. Show that the Fermi energy
$$\epsilon_F = \frac{h^2}{8m} \left(\frac{3N}{\pi V} \right)^{2/3}$$
for a gas of N non-interacting and non-relativistic electrons. Also, derive the integral expression that, if evaluated, (implicitly) determines the chemical potential of the gas at *any* temperature T.

    Hint: The energy of a single particle in a 3D box, with sides L, is given by: $E = \frac{\hbar^2}{2m} \vec{k}^2 = \frac{\hbar^2 \pi^2}{2mL^2} \vec{n}^2$

21. The Helmholtz free energy is defined as $F \equiv U - TS$ and is a useful quantity for constant T processes. Give a physical explanation for the TS-term for systems at such a condition. Also, use the laws of thermodynamics, to show that F is at its minimum for a system in thermal equilibrium with a heat bath.

22. The *grand* partition function $\mathcal{Z}$ can be used to describe systems of non-interacting, indistinguishable particles where quantum statistics is of importance. In particular, we used it to derive the Fermi-Dirac and Bose-Einstein distributions. How does one define the system and the reservoir in this case? Also, show how the entropy and mean energy can be calculated from the grand partition function in general.

23. Show first the general expression $S = -k \sum_i p_i \ln p_i$. Then, show that $\Phi_G = -kT \ln \mathcal{Z}$, where $\Phi_G = \bar{U} - \mu \bar{N} - TS$ is the grand potential and $\mathcal{Z}$ is the grand partition function. The expression for the grand canonical distribution function (p_i) can be used without proof.

24. Explain the following concepts
    - The thermodynamic limit
    - Weakly coupled systems
    - Entropy of mixing

25. Explain the concept of an Einstein solid. Also, show that $S \approx Nk_B \left[\ln \left(\frac{q}{N} \right) + 1 \right]$, when $N \gg 1$, $q \gg 1$ and $q \gg N$.

26. - Sketch the Fermi-Dirac distribution associated with a Fermi-gas at low (non-zero) T. Then, describe in words (and possibly formulas) how you would compute the chemical potential of such a gas, including what kind of approximations you make. Note, you do *not* need to do the actual mathematical calculation.
    - Finally, discuss Rayleigh's idea of how to obtain the spectral density of radiation for a black body.

27. Discuss, in detail, the connection between the partition function Z of a composite system and the partition functions Z_i associated with its different sub-systems i. In particular, how would you write Z for N non-interacting particles? Is there any difference in Z between classical and quantum particles? Give at least one other example (apart from N non-interacting particles) where we have used the idea of a composite system to construct Z in the course.

Answers to the questions can be found in the textbook and/or in the lecture notes. You will be asked to answer four of the questions as a part of the exam.