Formelblad Atom- och Kärnfysik

Rydberg:

$$\tilde{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)Z^2$$

$$E_n = -hcR\frac{Z^2}{n^2}$$

$$hcR_\infty = \frac{m_e(e^2/4\pi\epsilon_0)^2}{2\hbar^2} = 13,606 \text{ eV}$$

$$R = R_\infty \cdot \frac{M_N}{m_e + M_N} \qquad \text{(masskorr.)}$$

Alkalilika system, med $n^* = n - \delta_l$:

$$E = -hcR_\infty \frac{(Z_{eff})^2}{n^{*2}}$$

$$\Delta E_{FS} = \frac{Z_i^2 Z_o^2}{(n^*)^3 l(l+1)}\alpha^2 hcR_\infty$$

Vätelika atomer:

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r} = -\frac{Ze_0^2}{r}$$

$$a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2}$$

(I Bohrs atommodell: $r_n = a_0 n^2/Z$)

Radialfunktioner, $R_{n,l} = \frac{P_{n,l}}{r}$, för vätelika system:

$$R_{1,0} = \left(\frac{Z}{a_0}\right)^{3/2} 2e^{-Zr/a_0}$$

$$R_{2,0} = \left(\frac{Z}{2a_0}\right)^{3/2} 2(1 - \tfrac{Zr}{2a_0})e^{-Zr/2a_0}$$

$$R_{2,1} = \left(\frac{Z}{2a_0}\right)^{3/2} \tfrac{2}{\sqrt{3}}\tfrac{Zr}{2a_0}e^{-Zr/2a_0}$$

Klotytefunktioner $Y_l^m = Y_{l,m}$

$$Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\varphi}$$

$$Y_2^0 = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$$

$$Y_2^{\pm 1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\varphi}$$

$$Y_2^{\pm 2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\varphi}$$

Hamiltonoperator för flerelektronsystem:

$$\boldsymbol{H} = \sum_{i=1}^{N}\Big(-\frac{\hbar^2}{2m}\nabla_i^2 - \frac{Ze^2/4\pi\epsilon_0}{r_i} + \sum_{j>i}^{N}\frac{e^2/4\pi\epsilon_0}{r_{ij}}\Big)$$

$$\langle LM_L|\boldsymbol{l}_1|LM_L\rangle = \frac{\langle \boldsymbol{l}_1 \cdot \boldsymbol{L}\rangle}{L(L+1)}\langle LM_L|\boldsymbol{L}|LM_L\rangle$$

LS-koppling:

$$\text{termer} \quad \begin{cases} L = |l_1 - l_2|, \ldots, l_1 + l_2 \\ S = |s_1 - s_2|, \ldots, s_1 + s_2 \end{cases}$$

$$\text{nivåer} \quad J = |L - S|, \ldots, L + S$$

Zeemaneffekt:

$$E_{ZE} = \begin{cases} g_J\mu_B B M_J & \text{(end. finstruktur)} \\ g_F\mu_B B M_F & \text{(svagt fält, hfs)} \\ g_J\mu_B B M_J + AM_I M_J & \text{(starkt fält, } \mu_B B > A) \end{cases}$$

Koppling magnetiskt moment↔rörelsemängdsmoment:

$$g_S = 2$$

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

$$g_F = g_J\frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$$

$$\boldsymbol{\mu}_I = g_I\mu_N\boldsymbol{I}$$

Dopplerbredd:

$$\frac{\Delta\omega_D}{\omega_0} = 2\sqrt{\ln 2}\frac{u}{c} \approx 1,7\frac{u}{c}$$

$$u = 2230\sqrt{\frac{T}{300M}} \quad \text{m/s}$$

Hyperfinstruktur: $H = -\boldsymbol{\mu}_I \cdot \boldsymbol{B}_e = A\boldsymbol{I}\cdot\boldsymbol{J}$
För s-elektroner i vätelika system gäller:

$$A = \frac{2}{3}\mu_0 g_S\mu_B g_I\mu_N\frac{Z^3}{\pi a_0^3 n^3}$$

Boltzmanfördelningen:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-\Delta E/(kT)}$$

Harmoniska oscillatorn:

$$\phi_n = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\cdot H_n(\xi)e^{-\xi^2/2}, \quad \xi = \sqrt{\frac{m\omega}{\hbar}}x$$

$$E_n = \hbar\omega(n + \frac{1}{2})$$

$$H_0(\xi) = 1$$

$$H_1(\xi) = 2\xi$$

$$H_2(\xi) = -2 + 4\xi^2$$

$$H_3(\xi) = -12\xi + 8\xi^3$$

$$H_{n+1}(\xi) = 2\xi H_n(\xi) - H_n'(\xi)$$

Integraler

$$\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$$
$$\int_0^\infty x^{2n+1} e^{-\alpha x^2} dx = \frac{n!}{2\alpha^{n+1}}$$
$$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$
$$\int_0^\infty x^{2n} e^{-\alpha x^2} dx = \frac{(2n-1)!!}{2(2\alpha)^n}\sqrt{\frac{\pi}{\alpha}}$$

Operatorer

$$\boldsymbol{p} = -i\hbar\nabla$$
$$\boldsymbol{L} = -i\hbar\boldsymbol{r} \times \nabla$$
$$\boldsymbol{H} = -\frac{\hbar^2}{2m}\nabla^2 + V \quad \text{(standard)}$$
$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2} r + \frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2} + \frac{1}{\tan\theta}\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)$$

Diracnotation:

$$< \boldsymbol{H} > = < \psi|\boldsymbol{H}|\psi > = \int_{\mathbb{R}} \psi^* \boldsymbol{H}\psi dv$$

Kommutatorer:

$$[A,B] = AB - BA$$
$$[A,B] = -[B,A]$$
$$[A,B+C] = [A,B] + [A,C]$$
$$[AB,C] = A[B,C] + [A,C]B$$

Schrödingerekvationen:

$$\boldsymbol{H}\psi = E\psi \quad \text{(tidsober.)}$$
$$\boldsymbol{H}\psi = i\hbar\frac{\partial}{\partial t}\psi \quad \text{(tidsber.)}$$

Konfiguration	$\prod n_i l_i^{w_i}$
Termer	L och S (^{2S+1}L)
Nivåer	J
Tillstånd (ZE-subnivåer)	M_J
Hyperfinivåer	F

1	$\Delta J = 0, \pm 1$	$(J = 0 \not\leftrightarrow J' = 0)$	nivå
2	$\Delta M_J = 0, \pm 1$	$(M_J = 0 \not\leftrightarrow M_{j'} = 0$ om $\Delta J = 0)$	tillstånd
3	byt paritet		konfiguration
4	$\Delta l = \pm 1$		
5	$\Delta L = 0, \pm 1$	$(L = 0 \not\leftrightarrow L' = 0)$	term
6	$\Delta S = 0$		term

5, 6 bara om L och S är goda kvanttal.

	finstruktur - LS	hyperfinstruktur - IJ
växelverkan	$\beta\boldsymbol{L}\cdot\boldsymbol{S}$	$A\boldsymbol{I}\cdot\boldsymbol{J}$
moment	$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$	$\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{J}$
egentillstånd	$\|LSJM_J\rangle$	$\|IJFM_F\rangle$
energi	$\beta/2\big(J(J+1) - L(L+1) - S(S+1)\big)$	$A/2\big(F(F+1) - I(I+1) - J(J+1)\big)$
intervall	$E_J - E_{J-1} = \beta J$	$E_F - E_{F-1} = AF$
	(om $E_{S-O} \ll E_{re}$)	(om $A \gg \Delta E_{kvadrupol}$)

Kärnans uppbyggnad

Storlek: $R = r_0 \cdot A^{1/3}$

Q-värdet: $Q = m_a c^2 + m_A c^2 - m_b c^2 - m_B c^2$

$Q>0$ exoterm reaktion

$Q<0$ endoterm reaktion

Semiempiriska Massformeln

$$M(Z,N) = ZM(^1H) + Nm_n - B(Z,A)/c^2$$

$$B = a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{|A-2Z|^2}{A} + \delta$$

$\delta = +a_P/A^{3/4}$ för Z och N jämna

$\delta = 0$ för A udda

$\delta = -a_P/A^{3/4}$ för Z och N udda

Term	Värde (MeV)
a_V	15,5
a_S	16,8
a_C	0,72
a_{sym}	23
a_P	34

Relativitetsteori

$$E^2 = (pc)^2 + (m_0 c^2)^2$$

$$p = m \cdot v$$

$$T = E - m_0 \cdot c^2$$

$$m = \frac{m_0}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{v}{c}$$

Strålningsväxelverkan

Bethe-Blocks formel:

$$\frac{dE}{dx} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{4\pi z^2 ZN}{m_e v^2} \ln\left(\frac{2m_e v^2}{I}\right)$$

Comptonspridning:

$$E_\gamma' = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2}(1-\cos\Theta)}$$

Partikelenergi från cyklotron:

$$T = \frac{(ZeB)^2 \cdot R^2}{2 \cdot m}$$

Radioaktivitet

Sönderfallslagen: $dn = -\lambda n dt,\ n = n_0 \cdot e^{-\lambda t}$

Sönderfallskonstanten: λ

Aktiviteten : $A = \lambda \cdot n$

Halveringstid: $t_{1/2} = \frac{\ln 2}{\lambda}$

Seriesönderfall:

$$n_b(t) = \frac{n_{a0}\lambda_a}{\lambda_b - \lambda_a}(e^{-\lambda_a t} - e^{-\lambda_b t})$$

om $n_b(0) = 0$

Geiger-Nuttals lag: $-\log t_{1/2} = a + b \log E$

Q-värden i β-sönderfall:

$$Q_{\beta^-} = (M_X - M_Y)c^2$$

$$Q_{\beta^+} = (M_X - M_Y - 2m_e)c^2$$

$$Q_{EC} = M_X c^2 - M_Y c^2 - B_e(K)$$

Deexcitation:

$E_\gamma = E_i - E_f$

$T_e = E_{exc} - B_e - T_R$

Kärnreaktioner

Notation: a+A → b+B eller A(a,b)B

Tröskelenergi: $E_{tr} = -Q \cdot \frac{\sum m_i + \sum m_f}{2 \cdot m_{målkärna}}$

Attenuering: $I = I_0 e^{-N\sigma x}$

Rutherfordspridning:

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{4\pi\varepsilon_0}\right)^2 \cdot \left(\frac{1}{4T_a}\right)^2 \cdot \frac{1}{\sin^4(\theta/2)}$$

Neutronfysik:

$$\frac{T}{T'} = \frac{(A+1)^2}{A^2 + 1 + 2A\cos(\theta)}$$

$$\zeta = 1 + \frac{(A-1)^2}{2A}\ln\left(\frac{A-1}{A+1}\right)$$

och speciellt för $A = 1$ *är* $\zeta = 1$

$\Sigma = \sigma N$

Nedbromsningsförmågan: $S = \zeta \Sigma_s$

Moderationsförhållandet:

$$M = S/(N \cdot \sigma_a) = \zeta\Sigma_s/\Sigma_a = \zeta(\sigma_s/\sigma_a)$$