# Answers to problems; EXAMINATION IN CHAOS

**2007-01-11, 8–13**

4. $0 \leq r \leq \frac{3\sqrt{3}}{2}$, Fixed points: $x = 0$, $x = \sqrt{1 - 1/r}$. Bifurcation from one-periodic to two-periodic: $r = 2$.

5.

$$D_B = 1 + \frac{\log(L_1) - \log(L_2)}{\log(\delta_2) - \log(\delta_1)} \approx 1 + \frac{3.5 - 3.33}{2.75 - 1.60} \approx 1.15$$

6. a) $x_1 = x$, $x_2 = \dot{x}$, $x_3 = t$.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1^5 - 0.08x_2 + A\cos(x_3) \\ \dot{x}_3 &= 1 \end{aligned}$$

   Lie derivative: $-0.08 \Longrightarrow$ dissipative.
   b) Yes: Non-linear ($x^5$ term), dissipative, 3-dim flow
   c) $\lambda_1 + \lambda_2 = -0.08 \Longrightarrow \lambda_1 = 0.04$, one positive Lyapunov exponent means that the system is chaotic.

7. a)

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{4}{3}x^3 - \frac{1}{3}y^3$$

   b)

$$\begin{aligned} \dot{x} &= p_x/m \\ \dot{y} &= p_y/m \\ \dot{p}_x &= -4x^2 \\ \dot{p}_y &= y^2 \end{aligned}$$

   c) The system is separable and thus integrable