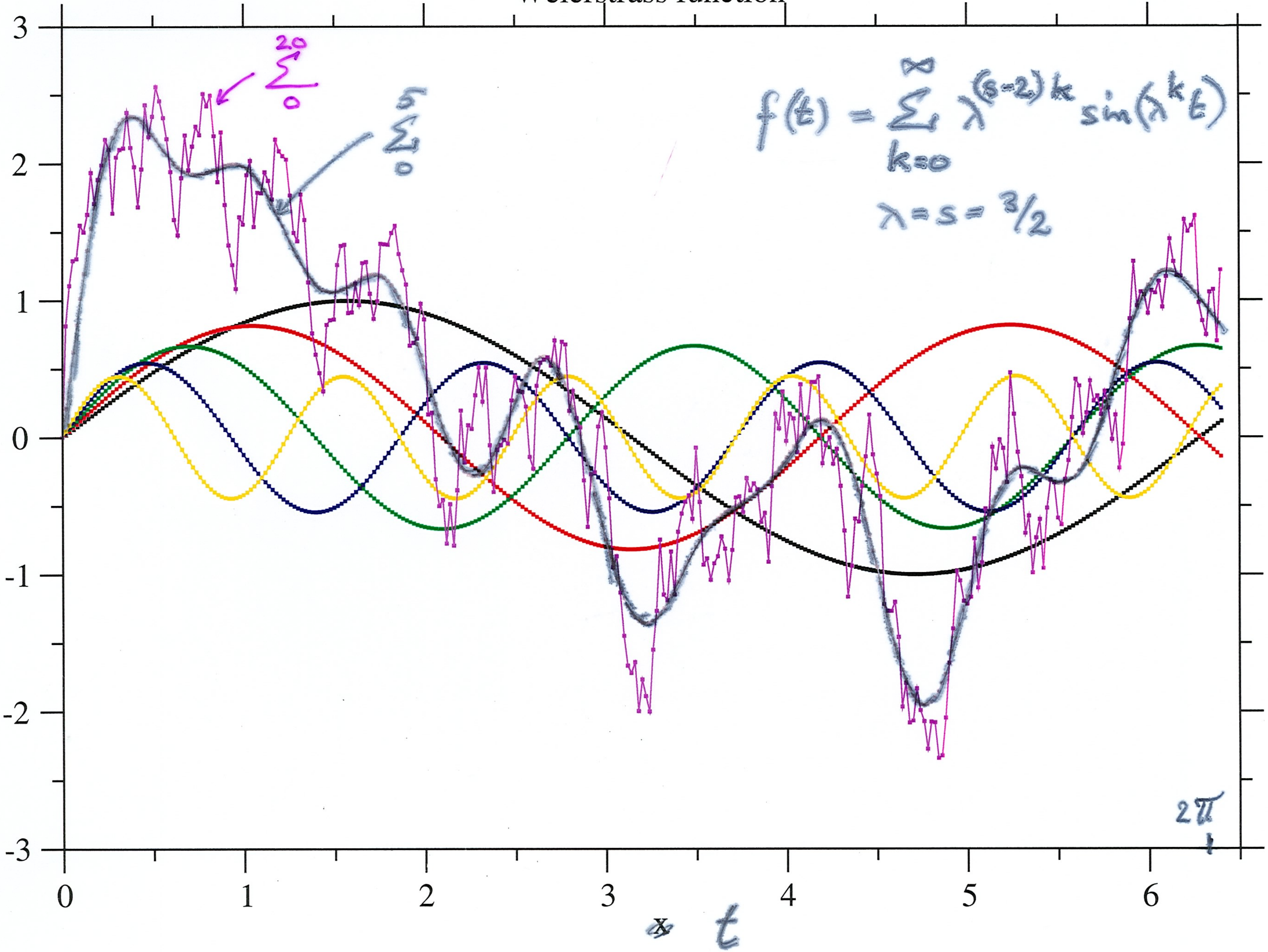
The first page shows the five first functions in the sum for the Weierstrass function, then the sum of these five functions and finally the sum of the 20 first functions.

The next page defines the devil's staircase and illustrates the first steps when constructing it.

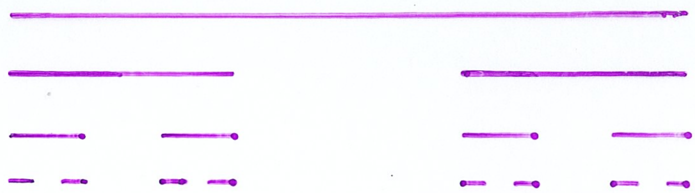
The last pages illustrates multiplication in the complex plane, where the last picture shows how to understand the 3-periodic points of $f_0(z) = z^2$.

Weierstrass function



Devil's Staircase

Start from Cantor set.



After n steps: 2^n intervals of length $(1/3)^n$

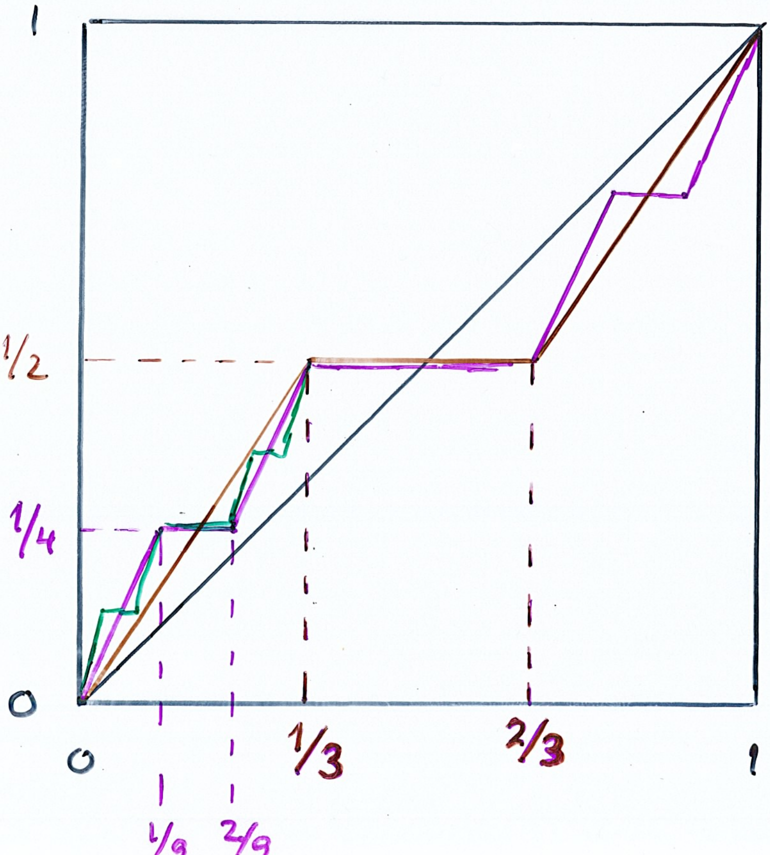
Number of intervals: 2^n

Total length: $(2/3)^n$

Total mass: 1

$$\rho_n(x) = \begin{cases} (3/2)^n & \text{if } x \in F_n \\ 0 & \text{if } x \notin F_n \end{cases}$$

$$D_n(x) = \int_0^x \rho_n(x')\,dx' \; ; \qquad D(x) = \lim_{n\to\infty} D_n(x)$$



Consider the map : $\boxed{f_c(z) = z^2 + c}$

Equivalent to logistic map extended to the complex plane.

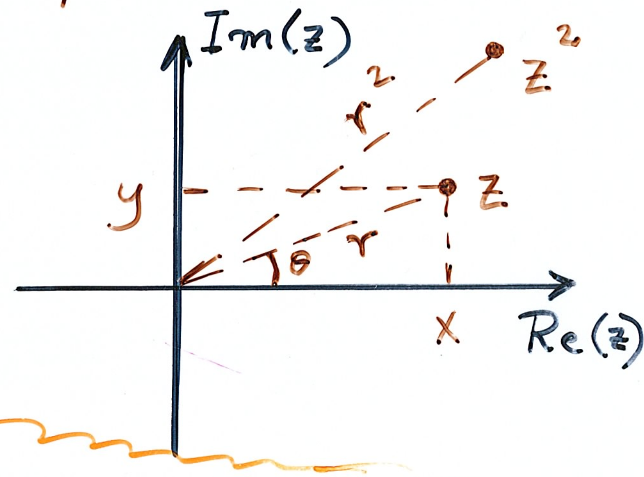
"Easy" to understand only for $c = 0$.

i.e. $\boxed{z_{j+1} = z_j^2}$

Understanding of z^2 for complex number

$$z = x + iy = re^{i\theta}$$

$$z^2 = r^2 e^{i2\theta}$$



If $r > 1$:

$$|z^n| = |r^n e^{in\theta}| = r^n \to \infty,$$

$$n \to \infty$$

$r < 1$

$$|z^n| \to 0, \quad n \to \infty$$

$r = 1$:

$$|z^n| = 1 \text{ for all } n$$

Thus : z^n is located on the unit circle for all n.

Consider the map: $z_{j+1} = z_j^2$

i.e. $f_0(z) = z^2$

$$\Rightarrow f_0^{(n)}(z) = z^{2^n}$$

① If $|z(0)| < 1 \Rightarrow \lim f_0^{(n)}(z_0) = 0$

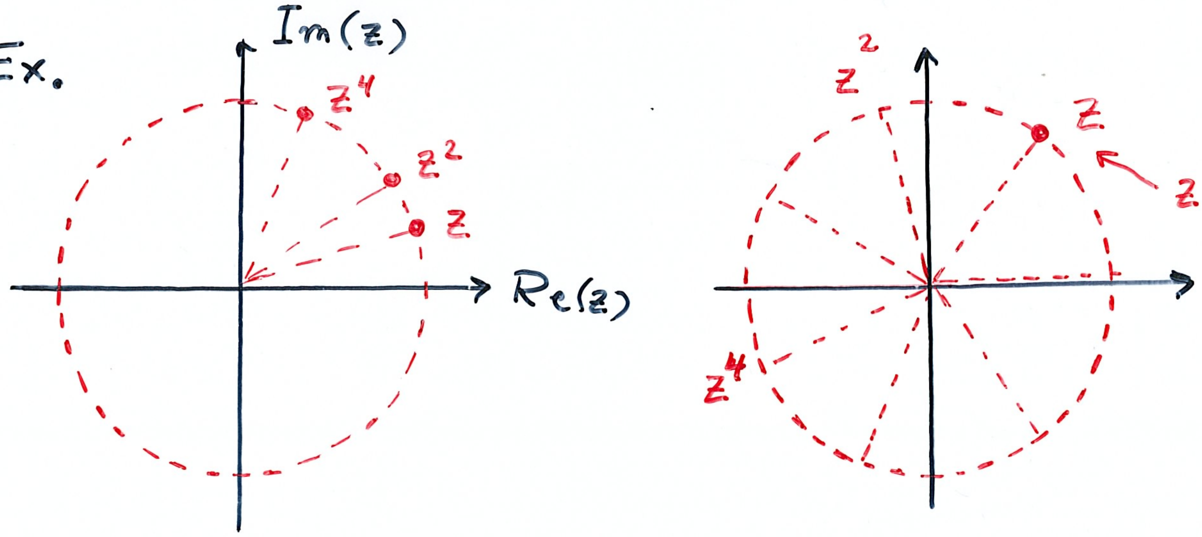
Origin (attractor with basin of attraction for all z with $|z| < 1$

② If $|z(0)| > 1 \Rightarrow \lim f_0^{(n)}(z_0) = \infty$

Infinity attractor with basin of attraction; all z with $|z| > 1$

③ If $|z_0| = 1 \Rightarrow f_0^{(n)}(z_0)$ stays on unit circle for all n.

Ex.



Conjecture: Infinite many periodic points on unit circle.