

1. How many groups are there of order 12?
2. Consider the full symmetry group of a regular cube. Reflections included.
 - a. How many elements does the group have?
 - b. Which are the subgroups?
 - c. Are there any invariant subgroups?
 - d. Which are the classes?
 - e. Work out the irreducible representations.
 - f. Construct the table of characters.
3. A methane molecule (CH_4) has four hydrogen atoms at the corners of a regular tetrahedron and a carbon atom in its center. Set up Newton's equation for the methane molecule assuming harmonic forces between the atoms. The spring constant is not the same between the hydrogens as between them and the carbon. Find the eigenmodes of the molecule.
4. An Ammonia molecule (NH_3) has three hydrogen atoms at the corners of a unilateral triangle and one Nitrogen atom at the center of the triangle some distance above its plane. Put appropriate s- and p-orbitals at each atom and set up the one-electron Schrödinger problem for the system. Construct the basis functions which transform as the rows of the irreducible representations of the symmetry group. Taking full advantage of group theory, which is the smallest secular problem that you have to solve in order to find all one-electron eigenvalues?
5. The group $\text{SU}(3)$ is defined as the group of linear unitary unimodular transformations in three dimensions. Unimodular means that their determinant is unity. How many generators does this group possess? Form the Lie algebra of the group.
 The isotropic harmonic oscillator in three dimensions is spherically symmetric. Yet, the eigenvalues show a much larger degree of degeneracy than can be expected from this simple fact. The extra degeneracy is caused by additional operators commuting with the Hamiltonian. Show that the operators

$$L_i \quad \text{and} \quad p_i p_j + w^2 x_i x_j$$
 form a set of symmetry operators which can be linearly combined so as to reproduce the Lie algebra of $\text{SU}(3)$. w is the frequency of the oscillator, $m = \hbar = 1$, x_i , p_i , and L_i are the i :th components of position, momentum and angular momentum respectively, and $i, j = x, y, z$. Try to find a ladder operator which increases the value of angular momentum squared.
6. In the one-electron approximation of the Nitrogen atom the ground state configuration is $1s^2, 2s^2, 2p^3$. Neglecting the spin-orbit interaction, work out the energies of the different allowed terms in the LS coupling scheme. What is the character of the lowest term?
7. The ground state of the Neon atom is a $1s^2, 2s^2, 2p^6$ configuration. A series of excited states are obtained by promoting one of the 2p electrons to the 3p shell. Which are the energies of the terms resulting from this configuration? Use LS coupling. Neglect spin-orbit interaction.