Lunds Universitet
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2015-11-02

Exercises to Electrodynamics, Week 1

(Homework to be handed in on Nov. 10 in Yousefs mailbox)

Exercise 1: *Electric fields*

Are the following vector fields possible static electric fields? Evaluate the charge density!

$$\mathbf{E}(\mathbf{r}) = r\mathbf{e}_x\,, \quad \mathbf{E}(\mathbf{r}) = f(r)\mathbf{r} \quad \text{with} \quad r = |\mathbf{r}|$$

Exercise 2: *Continuity equation*

a) Describe the following current densities and evaluate the temporal change in charge density

$$\mathbf{j}(\mathbf{r}) = \frac{I}{r^3}\mathbf{r}\,, \quad \mathbf{j}(\mathbf{r}) = f(r)\mathbf{e}_z \times \mathbf{r} \quad \text{with} \quad r = |\mathbf{r}|$$

b) Consider an infinitely long cylinder, centered around the z-axis and radius R. Evaluate the current through its surface for both current densities of a).

c) Compare the result of b) with a calculation via Gauss's theorem (or divergence theorem).

Exercise 3: *Circular polarized wave*

Formulate a circular polarized electromagnetic wave in complex notation.

Exercise 4 (Homework): *Helmholtz-coil*

a) A circular metallic ring (of infinitesimal thickness) of radius R has the z axis as symmetry axis. It carries a current I. Evaluate the magnetic field $B(x = 0, y = 0, z)$ on the z-axis.

b) A Helmholtz coil consists of two such parallel rings located at $z = -d/2$ and $z = d/2$. For which distance d is the magnetic induction as homogeneous as possible around $z = 0$? (i.e. the second order term in the Taylor expansion of $B(z)$ vanishes)

Exercise 5 (Homework): *Generator*

Consider a wire in the shape of a square loop with length l, which is rotating around the middle axis (x axis) with the angular frequency ω. At time $t = 0$ the loop lies in the xy-plane. There is a constant homogeneous magnetic field B in the z direction.

a) Evaluate the induced electromotive force for the closed circuit.

b) A lamp with resistance R is included in the circuit. Evaluate the current through the wire and the power used by the lamp using the electromotive force of a).

c) Evaluate the torque $\mathbf{N}_{\text{magnetic}} = \mathbf{m} \times \mathbf{B}$ on the loop. Here the current I provides the magnetic moment $\mathbf{m} = l^2 I\mathbf{n}$, where $\mathbf{n}$ is normal to the loop and the direction is given by the right hand rule. Which mechanical power $\omega\mathbf{e}_x \cdot \mathbf{N}_{\text{mech}}$ is needed to keep the loop going (i.e. $\mathbf{N}_{\text{magnetic}} + \mathbf{N}_{\text{mech}} = 0$)?

d) Discuss: Following the argument given above, the torque should become infinite for vanishing R. What is wrong?