Lunds Universitet
Andreas Wacker, Yousef, Yousefi 2015-11-9

Exercises to Electrodynamics, Week 2

(Homework to be handed in on Nov. 17 in Yousefs mailbox)

Exercise 1: *Plate capacitor*

A simple capacitor is a device formed by two insulated conductors adjacent to each other. The capacitance C can be defined by the capacitance C_{ii} of one conductor.
a) Does it matter, which conductor i is used?
b) Evaluate C approximatively for two parallel plates with distance d and area $A \gg d^2$.
c) Determine the force acting on a capacitor plate based on the electric field. (Assume a finite thickness for the surface charge and take into account, that the field drops to zero over this width!)
d) Evaluate the energy stored in the capacitor.
e) Show that the derivative of the energy with respect to d provides the same force as c). [If you keep the voltage fixed, you must take into account that charges flows in the circuit during this process.]

Exercise 2: *Vector potentials*

Show, that the following vector potentials

$$\begin{aligned}\mathbf{A}_1(\mathbf{r},t) &= 2(ct\mathbf{e}_x - ct\mathbf{e}_y + (ay - bx)\mathbf{e}_z)\\ \mathbf{A}_2(\mathbf{r},t) &= (b^2x^3 + abxy^2 + bz)\mathbf{e}_x + (abx^2y + a^2y^3 - az)\mathbf{e}_y + (ay - bx + ct)\mathbf{e}_z\end{aligned}$$

with $a, b, c \neq 0$ yield the same magnetic field $\mathbf{B}$. Find the gauge transformation, which transfers $\mathbf{A}_1$ to $\mathbf{A}_2$ and evaluate $\phi_2(\mathbf{r},t) - \phi_1(\mathbf{r},t)$ for the belonging scalar potentials.

Exercise 3 (Homework): *Screened Coulomb potential*

Consider the electrostatic potential $\phi(\mathbf{r}) = \frac{Q e^{-\mu r}}{4\pi\epsilon_0 r}$. Determine the charge density $\rho(\mathbf{r})$ and the total charge $\int d^3r\, \rho(\mathbf{r})$. Which physical situation does this correspond to?
Hints: For $\mathbf{r} \neq \mathbf{0}$, it is convenient to use spherical coordinates. To treat the singularity at $\mathbf{r} = \mathbf{0}$, use Gauss' theorem for the surface integral of the electric field.

Exercise 4 (Homework): *Quantum dot charging energy*

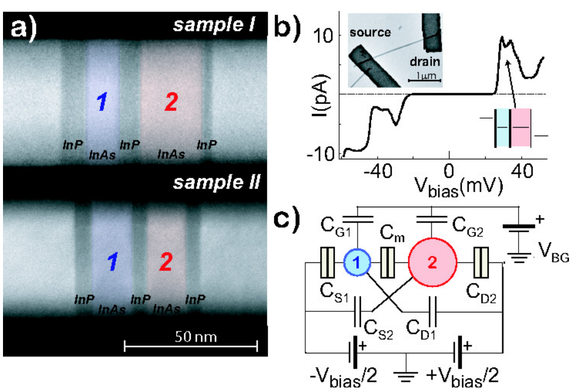

Fig.1 of Fuhrer *et al.*, Nanolett. **7**, 243 (2007). Note that the figure is not needed when solving the exercise.

In Lund, we can fabricate small semiconductor nanowires consisting of different semiconductor layers which can confine electrons in *quantum dots*, labeled by 1 and 2. In a typical experiment one can control the bias on the source (S, left), drain (D, right) and gate (G, from below) contact. One can approximate the source, drain, gate, and the dots as separate conductors and estimate the non-diagonal elements of the capacitance matrix by $C_{G1} = -1\text{aF}$, $C_{G2} = -3\text{aF}$, $C_{S1} = -8.5\text{aF}$, $C_{S2} = -2.5\text{aF}$, $C_{D1} = -0.8\text{aF}$, $C_{D2} = -2.8\text{aF}$, and $C_m = C_{12} = -3.4\text{aF}$. Evaluate the change in potentials ϕ_1, ϕ_2 for the respective dots if an additional electron enters dot 1, while the charge of dot 2 is unchanged and the potentials ϕ_S, ϕ_D, ϕ_G are fixed. Note that $C_{ij} = C_{ji}$ and $\sum_i C_{ji} = 0$ hold for the capacitance matrix. Also note that the entire capacitance matrix is not needed to solve this exercise.

Exercise 5 (Homework): *Potentials for the electromagnetic wave*

Determine the potentials $\mathbf{A}(\mathbf{r},t), \phi(\mathbf{r},t)$ for a planar electromagnetic wave in vacuum.