Quantum computation with superconducting qubits.

Ognjen Malkoc

June 10, 2013
Introduction

The implementation of error fault tolerant quantum information processing in a solid state scheme has been a desirable albeit difficult challenge. Compared to other approaches it would in principle enable a means of simply further extending circuit designs to incorporate also possible quantum computation. The difficulty in achieving a successful qubit resides in an uncompromising environment of a solid state system comprised of an amount of electrons approaching the thermodynamic limit. The necessity to ensure coherent controllability whilst protecting the states from vast source of unwanted interactions quickly makes clear that a clever design is necessary where the degree of interaction of a qubit can be controlled. Here the approach of superconducting circuit is introduced. Compared to other microscopic alternatives based on electrons, photons or ions, the quantum circuits are a collection of non-dissipative circuits and are comprised in a macroscopic design made possible by the phenomena of superconductivity and in particular the Josephson junction.

This report is structured as follows. Starting with a brief reminder on the workings of a Josephson junction which is subsequently used in the remainder of the report. The concept of light-matter interaction in cavities between atoms/superconducting qubits and their quantized electromagnetic background, is introduced in the section ‘QED in Circuits and Cavities’. In the section 'superconducting qubits' different types of design of the superconducting qubits are shortly reviewed in a chronological order before continuing to report on the progress that has been made utilising these designs, mainly based on [8]. It concludes with a short summary of possible advantages and disadvantages in view of other alternatives.

Superconductivity - Josephson junction

At very low temperatures electrons in some materials can experience an effectively attractive interaction where the total energy is reduced if two electrons near the Fermi level to pair up. This pair effectively becomes a new particle with total spin zero and can occupy the lowest energy state as it is not affected by the Pauli exclusion principle. The Fermi level changes and the process repeats. This results in the possibility for frictionless flow of a collective electron fluid in what is referred to as a ‘supercurrent’ as there is an excitation gap $\Delta$ because it costs energy to break a pair. Materials with this property are referred to as 'superconducting' materials which under a certain critical temperature $T_c$ make electrons pair up in cooper pairs.

An interesting effect takes place when connecting two superconducting electrodes via a thin strip of insulating material between them (sometimes called the weak link), through which electrons may tunnel, known as the Josephson effect. With the conditions that thermal excitations are far too low to break pairs ($kT \ll 2\Delta$) and that all electrons are paired up on both sides, the tunneling between the electrodes allows for cooper pairs to coherently tunnel from one electrode to the other. To study the consequences of this it is convenient to describe the states of the many body system by

$$ |m\rangle = |N_L - m, N_R + m\rangle$$

(1)
where the total amount of pairs $N = N_R + N_L$ is constant. The states are then characterised by $m$, the amount of pairs that are transferred from one side to the other. The tunneling Hamiltonian has the structure

$$H_T = -\frac{E_J}{2} \sum_m |m\rangle\langle m+1| + |m+1\rangle\langle m|$$

(2)

where $E_J$ is the tunneling energy. The two terms in the Hamiltonian correspond to tunneling from left to right and right to left respectively. Noting that the current operator $\hat{I} = 2e\frac{\hbar}{\pi}[H_T, \hat{m}]$ has eigenfunctions which are expressed in terms of $\phi$, the conjugate variable of $m$.

$$\hat{I}|\phi\rangle = I_c \sin \phi|\phi\rangle.$$  

(3)

Note the factor $2e$ due to the fact that pairs of electrons are tunneling. $I_c$ is the critical current, which is the maximum possible coherent current through the junction. Any greater current will break the condensate of cooper pairs. This relation is known as the first Josephson relation.

If there is a voltage drop $V$ over the junction, a new term must be added to the Hamiltonian

$$H = -E_J \cos \phi - (2e)V\hat{m}.$$  

(4)

From Hamilton’s equation of motion the time dependance of $\phi$ is then given by

$$\hbar \frac{\partial \phi}{\partial t} = 2eV, \quad \phi(t) = \phi(0) + \frac{2e}{\hbar}Vt.$$  

(5)

Since the current expression depends sinusoidally on $\phi$, we note that a regular DC voltage bias will lead to an oscillating current with the oscillation frequency $\frac{2e}{\hbar}V$. This is known as the second Josephson relation. For what will be relevant below the Josephson junction can be represented as a circuit where the flux describes the collective motion of the supercurrent. Due to the Heisenberg uncertainty relations, this superconducting circuit also allows for defining states which are e.g. superpositions of current traversing in opposite directions or positive and negative charges on the plates of a capacitor. Using these states the qubits are designed as being charge-based, flux-based or hybrids.

**QED in Circuits and Cavities**

**Cavity QED**

Cavity Quantum electrodynamics (CQED) deals with the interaction between single atoms and their electromagnetic background vacuum fluctuations. Normally such a coupling is far too low to enable practical coherent superposition states mixing the state of the atom and the background photons. There are two versions of CQED, optical and microwave.

In the optical version (shown in Fig 2.) an atom is sent through the cavity with a transit time of the atom is on the order of $\sim \mu s$.  

However, the lifetime of the atom is on the order of $\sim ns$ and detecting the state after the atom passes is not possible. Instead the optical transmission and spontaneous emission to modes which are not confined in the cavity are studied to obtain information about the state of the atom while traversing the cavity. In the microwave cavity approach a 3D superconducting resonator with a high Q-factor ($\sim$ lifetime of a resonant photon in the cavity) where one couples the photons in the cavity to a Rydberg atom.
The reason why Rydberg atoms are used can be heuristically explained by them being highly excited atoms, and as such the increasing distance between the ion and the valence electron result in a very large transition dipole moment. The life time of the atoms are on the order of $\sim \text{ms}$ and the state of the atom can be determined after the atom leaves the cavity. From this the state of the photons inside the cavity can be obtained. Both approaches are characterised by the resonance frequency of the cavity $\omega_r$, the transition frequency of the relevant two levels in the atom $\Omega = \omega_1 - \omega_0$ and the coupling of the photons in the cavity to the atom $g$. This system can be, in the presence of a single qubit, modelled by the Jaynes-Cummings type Hamiltonian

$$H_{JC} = H_0 + H_{int} + H_\gamma + H_\kappa.$$  \hspace{1cm} (6)

The last two terms concern the coupling of the atom states to the continuum of photons which are not part of the discrete set of modes in the cavity and the decay rate of the cavity respectively. The decay rate can be expressed in terms of the Q-factor and the resonance frequency. Since each discrete mode of the resonator may be treated as an independent harmonic oscillator with oscillator frequency $\omega_r$ and the spectrum of the appropriate atom can be restricted to two levels with level spacing $\hbar(\omega_1 - \omega_0)$ and can be treated as spin-1/2 particle

$$H_0 = \hbar\omega_r(a^\dagger a) + \frac{\hbar \Omega \sigma_z}{2}. \hspace{1cm} (7)$$

The electric modes of the resonator couple to the atom via its dipole moment. Restricting the interaction to only between one mode of the resonator will and the atom, the interaction can be written as

$$U = -\vec{p} \cdot \hat{\epsilon} E, \hspace{0.5cm} E = \hat{\epsilon} E_{ZPF}(a + a^\dagger) \hspace{1cm} (8)$$

where the quantized electric field $E$ is expressed in terms of the polarisation vector $\hat{\epsilon}$, the zero-point fluctuation amplitude $E_{ZPF}$ and photon creation (annihilation) operators $a^\dagger(a)$. The interaction Hamiltonian is therefore given by

$$H_{int} = \langle \Psi_1 | (-\vec{p} \cdot \hat{\epsilon}) | \Psi_0 \rangle E_{ZPF}(a + a^\dagger) \sigma_z = \hbar g (a^\dagger \sigma^+ + a \sigma^-) + \hbar g (a^\dagger \sigma^- + a \sigma^+) \hspace{1cm} (9)$$

where it has been used that $\hbar g = \langle \Psi_1 | (-\vec{p} \cdot \hat{\epsilon}) | \Psi_0 \rangle E_{ZPF}$ and that $\sigma_z = \sigma^+ + \sigma^-$. Here $|\Psi_0\rangle$ and $|\Psi_1\rangle$ refer to the ground state and the excited state of atom plus cavity respectively.

By dropping the last term one obtains the interaction part of the Jaynes-Cummings Hamiltonian. Usually this is referred to as the rotating wave approximation and is valid for $|\Omega - \omega_r| \ll \Omega + \omega_r$. An atom will, via the dipole moment, absorb a photon from the electromagnetic background or emit a photon due to a transition in the atom. This results in dressed states, i.e. superpositions of cavity excitations and atomic states.
In the regime of zero detuning ($\Omega = \omega_r$) this means that the degeneracy of the atom plus cavity states is lifted with a splitting which depends on the amount of excitations in the cavity (See left part of Fig. 3). Meanwhile in the dispersive regime ($g/(|\omega_r - \Omega|) \gg 1$) there occurs a shift to second order in $g$ which is independent of the amount of excitations but depends on the state of the atom. This means that for a strong coupling $g \gg \gamma, \kappa$ where the splitting is greater than the line width due to the decay processes of the dressed states (where both decay processes contribute as it is a mixture of photons and excited atoms), it is possible to determine the atomic state by studying how the atom effectively 'pulls' the cavity frequency.

The critical role of strong coupling regime has led to the development of architectures which employ 'artificial atoms' placed inside one dimensional transmission lines which function as cavities.

### Circuit QED

In circuit QED (cQED) the role of the atom is replaced by a superconducting qubit, which is placed inside the cavity. Moreover the fixed position of the qubit makes it possible to use also a one-dimensional structure like the 1D transmission line resonator as the cavity. The immediate benefit of this is the possibility of increasing the zero point fluctuations of the cavity field $E_{ZPF}$. This makes it easier to achieve a strong coupling regime where the system is once again described by just the Jaynes-Cummings Hamiltonian with the vacuum Rabi coupling [1].

$$g \sim \sqrt{\frac{\hbar \omega_r}{c L}}$$

where $L$ is the resonator length and $c$ is the capacitance per unit length of the transmission line.

This system is schematically illustrated in Fig 4. Another benefit in this scheme is the possibility to control coherence life time of the qubit. By placing the the qubit at a fixed position inside the cavity, reducing the effect of the spatial variations of the electric field in the cavity, it is possible to amplify the spontaneous emission of the atom via the Purcell effect (which states that the spontaneous emission can be enhanced by matching the transition frequency). By design spontaneous emission can be suppressed in the regime of a large detuning as the qubit couples directly to the cavity modes and only indirectly to the environment continuum.
Moreover with transmission line resonances a Q-factor on the order of $\sim 10^6$ has been demonstrated, limiting the linewidth due to cavity decay to orders of magnitude smaller than the Rabi splitting.

Similar to the case of CQED, the readout of the state is done in the dispersive regime. In this regime the qubit far detuned from the cavity ($g/|\omega_r - \Omega| \gg 1$) where diagonalizing the Hamiltonian gives to the lowest order

$$H_{\text{int}} = \frac{\hbar}{\Omega - \omega_r} (a^\dagger a + \frac{1}{2}) \sigma_z.$$  

(11)

This leads to a shift of the cavity frequency which depends on the qubit state and changes the photons which are transmitted through the cavity. What is important to note is that this gives a Quantum Non-Demolition (QND) measurement of both the photon number (cavity excitations) and qubit energy. While the most basic measurement would e.g. annihilate a photon and turn it into an electrical signal which is registered, a QND measurement does not affect the state measured and it becomes possible after measurements of an ensemble of states to obtain a distribution post-measurement which coincides with the distribution pre-measurement. With these measurements non-classical photons in these circuits have been observed [4] where the states were superpositions of one and zero excitations of the cavity and it was seen that the mean field of the electric field of the one photon fock state was zero. Here the amplitude and phase of the non-classical photon was also demonstrated to be controllable.

**Superconducting qubits**

The idea to introduce an artificial atom as a circuit has a crucial criterion, it should be restricted to non-dissipative circuits to protect coherence of the states. One such basic circuit is a regular LC-circuit. We can study the properties of such a circuit by studying the Lagrangian. If we were to consider the flux in the circuit to function as the coordinate of the system and use that $\Phi(t) = \int_{-\infty}^{t} d\tau V(\tau)$ we obtain

$$\mathcal{L} = T - V = \frac{C \dot{\Phi}^2}{2} - \frac{1}{2L} \Phi^2$$  

(12)

where $T$ is the kinetic and $V$ the potential energy in the LC circuit. From the Lagrangian we obtain that the conjugate variable of the flux is the charge on the capacitor

$$\frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C \dot{\Phi} = Q$$  

(13)

which after a legendre transformation gives a Hamiltonian for the system

$$H = Q \dot{\Phi} - \mathcal{L} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}.$$  

(14)

These two variables $\Phi, Q$ characterise the system and by making them operators that satisfy

$$[\hat{Q}, \hat{\Phi}] = i\hbar$$  

(15)

we obtain the Hamiltonian of the quantized LC circuit. These operators can in turn be expressed in terms of ladder operators $a = i\sqrt{\frac{1}{2C \hbar}} \hat{\Phi} + \sqrt{\frac{1}{2L \hbar}} \hat{Q}$ with the natural oscillation frequency
\[ \Omega = \frac{1}{\sqrt{LC}}. \] These operators are constructed such that \([a, a^\dagger] = 1\), and we see that we reproduce the energy spectrum of a harmonic oscillator

\[ H = \hbar\omega_{LC}(a^\dagger a + \frac{1}{2}), \quad \omega_{LC} = \sqrt{\frac{1}{LC}} \tag{16} \]

That is, the simplest non-dissipative circuit will result in an energy spectrum of equidistant levels. This makes it difficult to use such a circuit as a qubit as one cannot define a set of logic levels \([|0\rangle, |1\rangle]\) since the higher lying states are not sufficiently separated from these levels, ultimately removing the possibility to discriminate non-logic states. This is where the properties of a Josephson junction comes into play. In a circuit representation the Josephson junction can be represented as a non-dissipative circuit consisting of a capacitor and an inductor in parallel which is non-linear in the phase over the junction. The non-linearity enables a non-equidistant energy spectrum where the conjugate variables of the Josephson junction, \(\hat{N}, \hat{\Phi}\), are used to define the qubit states. This property has led to qubit designs based on circuits whose logic states are defined by either the charge (charge qubits) or flux (flux qubits).

### Cooper pair box

The simplest charge based qubit which was already considered in 1987 \[2\] is referred to as the Cooper pair box (CPB). The qubit is schematically shown in Fig. 5. The energy spectrum of the CPB are characterised by two contributing energies. The capacitive charging energy \(E_C\) which comes from the fact that there is a capacitance between left and right side of the junction. It can effectively be seen as the work necessary from transfer a cooper pair from one side to the other

\[ E_C = \frac{(2e)^2}{2C}. \tag{17} \]

The total operator describing the capacitive energy is then given by

\[ \hat{U} = E_C(\hat{n} - n_g). \tag{18} \]

While here transferred pairs, \(\hat{n}\), has integer eigenvalues, \(n_g\) is a continuous variable describing any effect that might lead to breaking the degeneracy of transferring pairs to both sides.

It can either be an externally applied electric field, an applied gate potential inducing an offset charge on the capacitor. Together with the tunneling term of a Josephson junction the total Hamiltonian of the CPB is in the phase representation \(|\phi\rangle\) given by

\[ H_{CPB} = E_C(\hat{n} - n_g)^2 - E_J \cos \phi. \tag{19} \]

From the equation of motion for \(\phi\) we can see that it is directly proportional to the flux variable

\[ \frac{\hbar \phi}{\pi} = 2eV, \quad \frac{\partial \Phi}{\partial t} = 2eV \quad \Rightarrow \quad \phi(t) = \frac{2e\Phi(t)}{\hbar} + K. \tag{20} \]
where K only contributes with a constant phase shift over the junction. The addition of a flux quanta ($\Phi_0 = \frac{h}{2e}$) consequently adds $2\pi$ to the superconducting phase over the junction. The total Hamiltonian then becomes

$$H_{CPB} = E_C(\hat{n} - n_g)^2 - E_J \cos \left( \frac{2\pi \Phi(t)}{\Phi_0} \right).$$  \hspace{1cm} (21)

From this expression the non-linearity of the CPB is seen when comparing with the Hamiltonian of the LC-oscillator

$$L_{LC} = \left( \frac{\partial^2 H_{LC}}{\partial \Phi^2} \right)^{-1} \rightarrow L_{CPB} = \left( \frac{\partial^2 H_{CPB}}{\partial \Phi^2} \right)^{-1} = \left( E_J \frac{2\pi}{\Phi_0} \cos(2\pi \Phi_0) \right)^{-1};$$  \hspace{1cm} (22)

To manipulate the Hamiltonian a gate potential is used to control the offset charge $n_g$. By varying the gate potential $V_g$ the offset charge $n_g = \frac{C_g V_g}{2e} + \delta n_g$ can be set to $n_g = 0.5$. Here $\delta n_g$ is the offset charge due to e.g. possible imperfections of the wires. The energy spectrum for this operational 'sweet spot' is illustrated in Fig. 5. At this point two lowest states $|n = 0\rangle$, $|n = 1\rangle$ are degenerate and can be practically used to define charge-based qubit states. Around this point the system behaves effectively as a two level system that can be represented as a 'pseudo-spin 1/2' system

$$H_{CPB} = E_C(n_g - 1/2)\sigma_z - 1/2E_J\sigma_x.$$  \hspace{1cm} (23)

The control parameter is then $E_J$. By coupling two (to simplify matters, identical) Josephson junctions in parallel, and constructing what is known as a DC superconducting interference device (DC-SQUID), it can be shown that the control parameter is made tunable via external flux $\Phi_{ext}$ through the two-junction loop, such that $E_J \rightarrow E_J(\Phi_{ext}) = 2E_J \cos \left( 2\pi \frac{\Phi_{ext}}{\Phi_0} \right)$.

However, depending on the offset charge parameter $n_g$ to enable a suitable qubit basis makes the cooper pair box sensitive to possible e.g. fluctuations of the electric field. This sensitivity may for instance be adjusted by designing the qubit with respect to the ratio $E_C/E_J$ and as such giving less weight to energies giving rise to larger fluctuations. The study of identifying sources of fluctuations, which cause decoherence in the system, has led to new qubit designs which by design suppress the influence of these fluctuations.

**Flux box**

A dual to the Cooper pair box design, called a flux box (FB) is illustrated in Fig. 6. It is based on the flux degree of freedom in the circuit rather than charge. It works by having the two sides of a Josephson junction with the capacitance $C_J$, connect by a superconducting loop with an inductance $L$. This loop coupled to an auxiliary coil through some mutual inductance. This will inductively shunt the Josephson junction as the auxiliary coil imposes in that way an external flux $\Phi_{ext}$ onto the superconducting qubit. The external flux then functions as the offset flux of the Josephson junction in the circuit similar to the offset charge of the CPB. This is the same workings as that of an RF-SQUID. The Hamiltonian of this quantised circuit can be written as

$$H_{FB} = \frac{(2e)^2}{2C_J} \hat{n}^2 - E_J \cos(\phi + \phi_g) + \frac{\phi^2}{2L}$$  \hspace{1cm} (24)

where here in contrast to the cooper pair box $\hat{n}$ does not have integer eigenvalues. By designing the qubit such that the relative strength $\frac{E_J}{E_C} \gg 1$ and let the inductance of the loop come close to the Josephson junction inductance $L_J/L \sim 1$ we can obtain, just like in the cooper pair box, an operating 'sweet spot' by flux biasing the qubit with $\Phi_{ext} = \Phi_0/2$.  \hspace{1cm} (24)
By working with a design where the capacitive energy is weighted less, the influence of charge fluctuations is naturally reduced. In Fig. 6 the resulting energy spectrum as a function of the flux is shown. The two lowest energy levels are given by the symmetric and anti-symmetric combinations of the wave functions inside both wells, and they can form the set of logic states. Near the ‘sweet point’ the Hamiltonian of the system becomes effectively a two-level qubit Hamiltonian again

$$H_{FB} = -\frac{E_S}{2} \sigma_Z + 2k \frac{E_L}{E_S} (1/2 - \frac{\Phi_{ext}}{\Phi_0})$$ (25)

where $E_S \sim \sqrt{E_B E_C} e^{-\xi \sqrt{E_B/E_C}}$ is the splitting energy arising due to the barrier height barrier height $E_B \sim E_J^2$ [7] in the energy spectrum and k, $\xi$ are constants determined numerically. The second term shows how, near the point $\frac{\Phi_{ext}}{\Phi_0} = 1/2$, the control of the qubit is done by flux biasing the circuit. This however also means that this design is sensitive to the presence of flux fluctuations.

### Current biased junction

Another design which has been considered is the current biased junction, a phase qubit. Using again $E_J/E_{C_J} \gg 1$ and biasing a Josephson junction with a current source results in a tilted washboard energy spectrum (see Fig. 7). Apart from the decreased sensitivity to charge fluctuations because of less weight to the capacitive energies, the potential profile also offers one advantage over the other designs. Because of the possible leakage by tunneling through the barrier there is already a readout mechanism. Using the lower states of each well as the logical states it is possible to determine the state of the qubit by measuring the occupation probability of the higher lying state.

### Transmon qubit

Recently the development of superconducting qubits has been to design the qubits with the available parameters $E_J, E_C$ to improve coherence time by attempting to make the states insensitive to known sources of noise. One such design is a charge-based qubit, whose Hamiltonian is identical to that of a cooper pair box, called the ‘Transmon qubit’. The principle behind the design is to reduce the sensitivity to charge fluctuations by noting that the n’th energy level as a function of any
potential offset charge $n_g$ can be approximated by noticing the similarity between the Hamiltonian of the cooper pair box and the tight-binding model [10]

$$E_n(n_g) = \varepsilon + \varepsilon_n(n_g) \cos(2\pi n_g), \quad \varepsilon_n(n_g) \sim (-1)^n E_C \frac{24m+5}{m!} \sqrt{\frac{2}{\pi}} \left( \frac{E_J}{2E_C} \right)^{\frac{m}{2}} e^{-\sqrt{8E_J/E_C}}$$  \quad (26)

It is seen here that the sensitivity to any possible charge fluctuations, which is described as fluctuations of $n_g$, is reduced for large $E_J/E_C$. However, the insensitivity to charge fluctuations is paid for by a decrease in anharmonicity (See Fig. 8) and it becomes a matter of balancing the dephasing time due to charge fluctuations and the minimum pulse duration which avoids transitions outside the two lowest energy levels which constitute the logic states of the qubit.

**Fluxonium qubit**

Building on the flux based qubit there has also recently been a development in what is referred to as the fluxonium qubit. In this design a Josephson junction is inductively shunted by a series of junctions to achieve an inductance $L_A$ much greater than the inductance of the Josephson junction circuit. This design is insensitive to offset charges while at the same time preserves an anharmonic energy spectrum. The circuit for this design is sketched in Fig. 9 along with the resulting energy spectrum.
Figure 9: Left: Fluxonium circuit. A Josephson junction with inductance $L_J$ and capacitance $C_J$ is shunted by an array of larger Josephson junctions with inductance $L_{JA}$ and capacitance $C_{JA}$. Each array element has a small capacitance to the ground $C_g$. Right: Anharmonic energy spectrum as a function of the phase $\varphi$. Image from [11].

Coupling qubits

In order to establish a computational basis consisting of multiple qubits, an interaction between the qubits has to be introduced. One possible approach which has been investigated is to couple the qubits directly to each other via means of mutual capacitance or a shared inductance.

Such alternatives can be seen in Fig. 10. By coupling the with a shared inductance [14] the effective Hamiltonian can with suitable gate voltage reduced to the form

$$H = \sum_{k=1,j} \left[ \varepsilon_k(V_{Xk})\sigma_z^k - \bar{E}_{Jk}\sigma_x^k \right] + \Pi_{ij}(\Phi_e, \Phi_{Xi}, \Phi_{Xj})\sigma_x^{(1)}\sigma_x^{(2)} \tag{27}$$

where the inter-qubit coupling can be controlled by the external and local fluxes $\Phi_e, \Phi_{Xi}, \Phi_{Xj}$. With this form it is clear that in principle two qubit operations such as the CNOT is made possible. An alternative approach, which is further described below, is to place the qubits within a cavity and couple the two qubits via the cavity mode. This also enables a decoupling of the qubits by tuning the cavity frequency. As an additional benefit it allows for a greater isolation of the qubits from noise that can reduce the phase coherence lifetime of the states.

Achievements

In 2009 it was reported that a group at Yale University together with groups at the University of Waterloo, Vienna University of Technology and Université de Sherbrooke managed to demonstrate two-qubit algorithms using a circuit QED architecture. To achieve this a circuit
was constructed where two transmon qubits couple to a transmission line cavity which is used to control and measure the qubit states. Using this design they were able to attain a coherence time for the qubits on the order of $\sim 1\mu s$, a sufficient amount of time to perform $\sim 10$ single gate operations. This has enabled realising simple algorithms with a success rate of over 80%.

They were able to tune the qubits off resonance with the cavity and each other where the coherence times of the qubits increased. At this point, referred to as ‘point I’ in Fig. 12, initialization of the computational basis, local single-qubit operations and measurements were done. Tuning the qubits on resonance with the cavity allowed them to extract the vacuum rabi oscillations and effectively determine the interaction strength with the cavity by studying anticrossing resulting from the splitting of the degeneracy. This is point II in Fig. 12. Moving from point I to this point it was possible to couple the two qubits to each other in a way that simulates a two-qubit phase gate

$$U = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & e^{i\phi_{01}} & 0 & 0 \\
0 & 0 & e^{i\phi_{10}} & 0 \\
0 & 0 & 0 & e^{i\phi_{11}}
\end{pmatrix},$$

(28)

where $\phi_{lr}$ is the dynamical phase acquired by state $|l, r\rangle$ going from point I to point II. As mentioned before, detuning the cavity to the strong-dispersive regime it is possible to obtain a joint readout. As this readout allows for performing QND measurements which are qubit-state dependent, they were able to study the two-qubit correlations.

Moreover, by tuning the cavity it is possible to obtain a conditional phase gate, a C-phase gate. This allowed them to generate any of the four bell states in the basis of the two qubits. Using this they were able to report a concurrence of $C > 0.8$ when preparing any of the four bell states. The concurrence $C$ is a measure of entanglement for two qubits which is zero for separable states (classical states) and 1 for maximally entangled states (i.e. any of the four Bell states.).

Aside from being able to generate entangled states, the C-phase gate also enables more complicated algorithms such as Grover’s search algorithms, described in Fig. 11 and the Deutsch-Josza algorithm. Combining single qubit operations with the two qubit C-phase gate they report a fidelity of 85% to obtain the right answer when performing Grover’s search algorithm.

In a different experiment by a group at École Normale Supérieure [5] using a superconducting circuit, a photon-mediated interaction was realised between two spatially separated quantum dots. With a separation distance 200 times the size of the dots all interactions due to direct capacitive and tunnelling couplings could be disregarded. This opens up the possibility for future scaling the cQED devices where new qubits could be introduced without being accompanied with noise due to inter-qubit coupling.
Conclusions

Where formerly quantum information processing has been dominated by other approaches such as trapped ions, the superconducting circuits have become increasingly popular. With the introduction of a cavity mediated interaction, coherent control of the qubits has improved significantly. This has been enabled by being able to construct circuits that by design reduce the influence of unwanted sources of noise. An increasing knowledge about possible decoherence sources together with the versatility in design of both the cavity and the superconducting qubits has led to experiments reporting an increase of performance by several magnitudes in the last decade. The circuits are now at a stage where they can be initialized, controlled and read within the coherence time of the qubit. As mentioned in the previous section two qubit operations have been made possible and robust enough to perform simple algorithms. Furthermore, the fact that it is an on-chip design which only requires existing lithographic techniques makes it a good candidate for a solid state based quantum information processor. However, while robustness to different noise sources has been demonstrated experimentally, this robustness has not been universal. Qubits based on e.g. the cooper pair box, such as the transmon, has managed to suppress some sources of decoherence but suffer from other less understood relaxation sources. Moreover, scalability of these circuits has not been explored fully and in terms of the amount of qubits it is far from what has been made possible with ion traps. At the same time the spawned interest in other parts of fundamental research such as Single-Photon detectors, detecting Hawking radiation in these circuits and observing the dynamical Casimir effect shows the remarkable precision and coherent controllability of the superconducting circuits. Despite its shortcomings, the rapid, almost systematic, advances made in terms of identifying and suppressing decoherence sources the last decade make the approach of superconducting circuits a promising candidate for processing quantum information.
Bibliography


