Macroscopic Quantum Tunneling and Coherence

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Set-up

Lecture 1: Basics and Theoretical Tools

Goal: How to describe quantum tunneling

Lecture 2: Tunneling in Macroscopic Systems ?

Goal: Understanding MQT and MQC

Lecture 3: Tunneling in Josephson Systems

Goal: Understanding MQT and MQC in Josephson devices

Set-up

Lecture 1: Basics and Theoretical Tools

- Introduction
- Simple examples
- Transmission and decay rates
- ImF-technique
- Quantum Coherence

Goal: How to describe quantum tunneling

Lecture 2: Tunneling in Macroscopic Systems ?

Lecture 3: Tunneling in Josephson Systems

Quantum tunneling is cool !

- 5 nobel prizes in physics related to tunneling (semiconductors, superconductors, STM)
- Quantum tunneling: Google about 2 million hits
- Macroscopic quantum tunneling: Google about 500.000 hits

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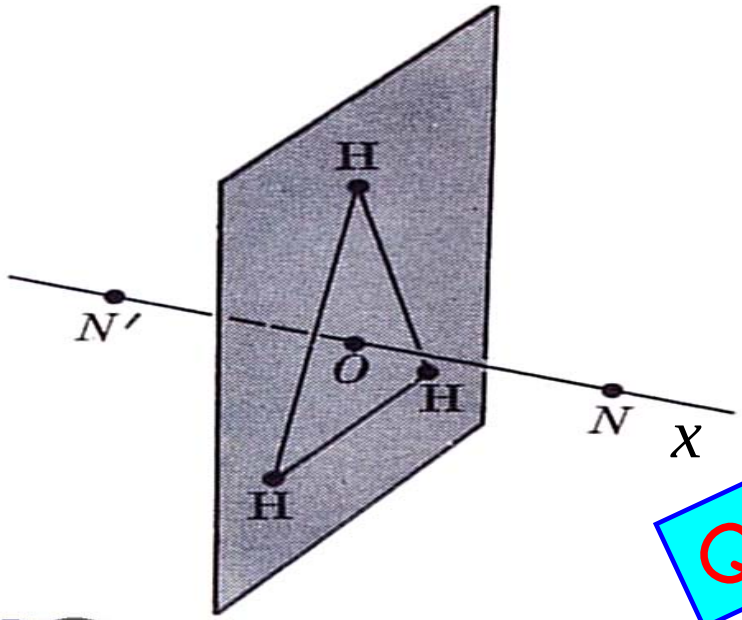
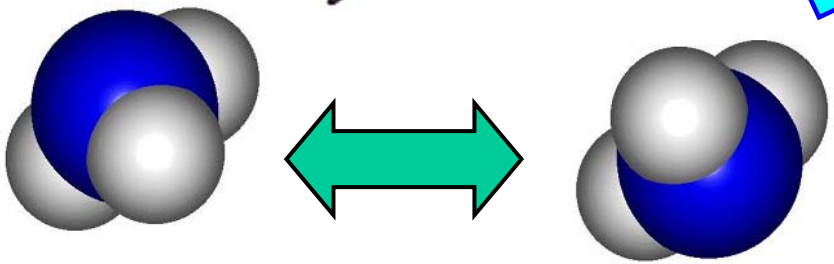
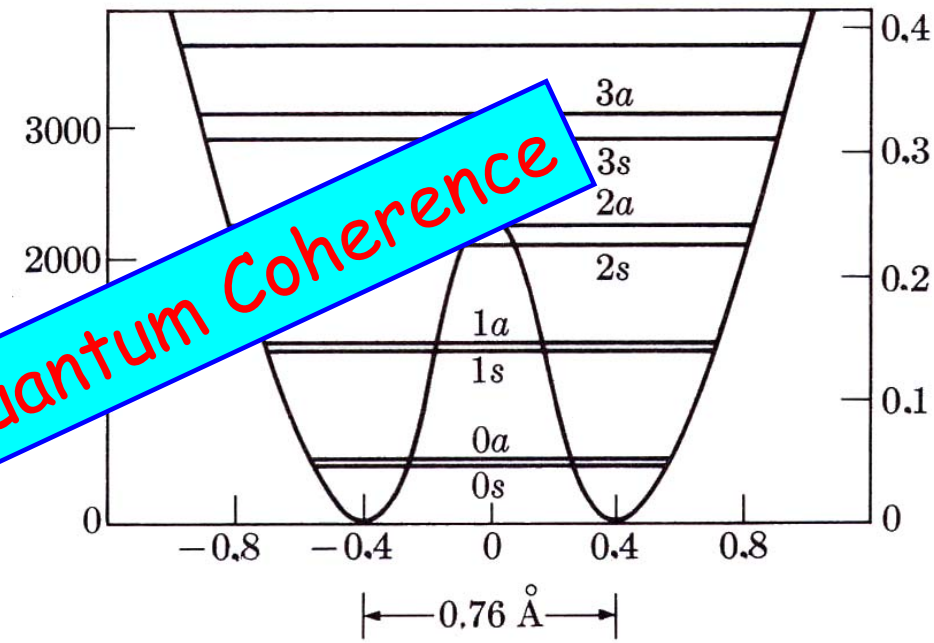
1923, De Broglie:
Matter waves, in analogy to optics:
penetration of forbidden regions

Tunneling in NH_3

Friedrich Hund 1927: „Zur Deutung der Molekülspektren"





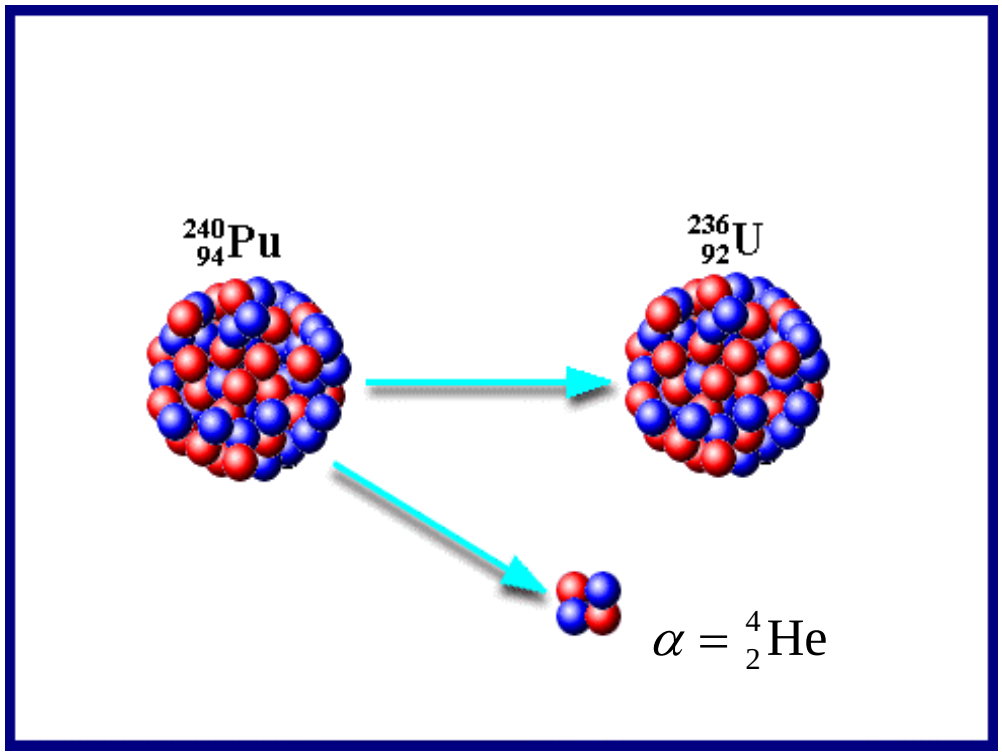


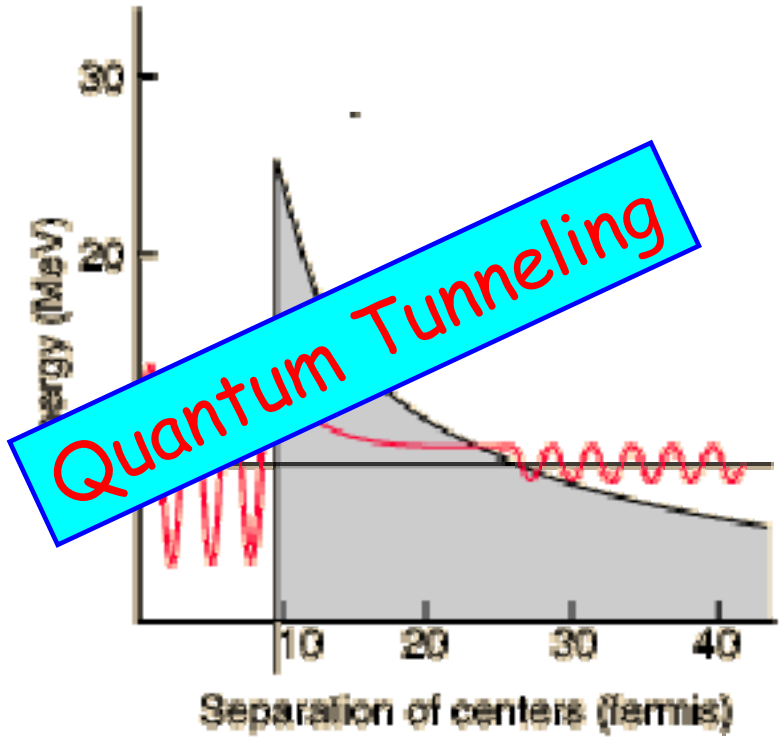
Quantum Coherence

F. Hund, Z. Phys. 40, 742 (1927) and 43, 805 (1927)

Alpha-Decay

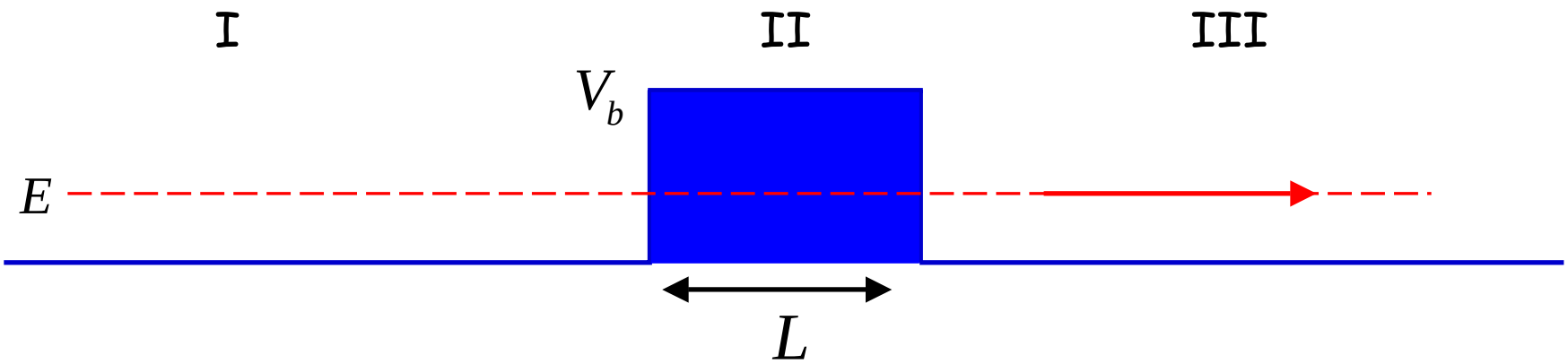
G. Gamow, Z. Phys. 51, 204 (1928)
J.R. Oppenheimer, Phys. Rev. 31, 80 (1928)
R.W. Gurney, E.U. Condon, Nature 122, 439 (1928)





$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y}^{*} + {}^{4}_{2}\mathrm{He} + E_{\alpha} $$

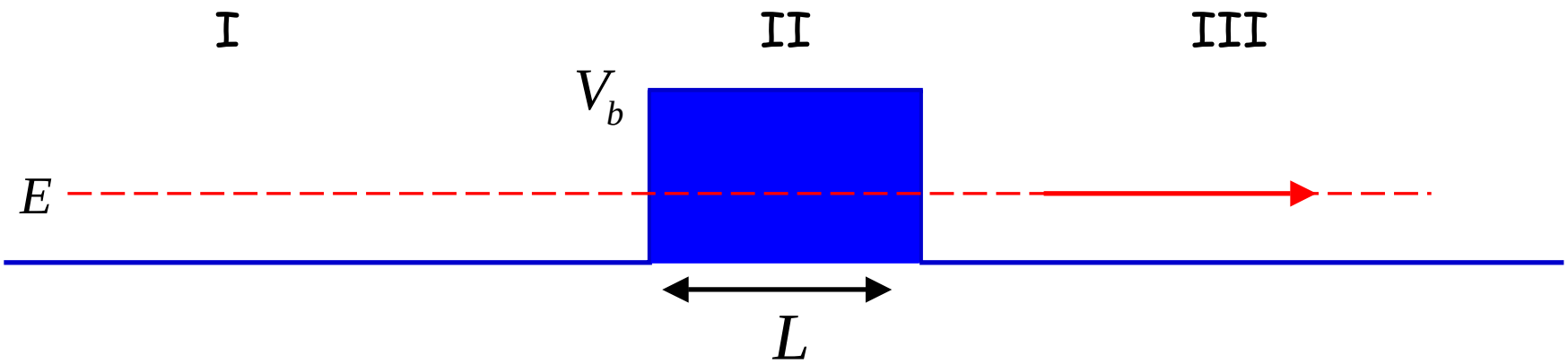
Barrier transmission: Scattering



$$
\begin{aligned}
\phi_I(x) &= \mathrm{e}^{ikx} + r(E)\,\mathrm{e}^{-ikx} \\
\phi_{II}(x) &= a\,\mathrm{e}^{-\kappa x} + b\,\mathrm{e}^{\kappa x} \\
\phi_{III}(x) &= t(E)\,\mathrm{e}^{ikx}
\end{aligned}
$$

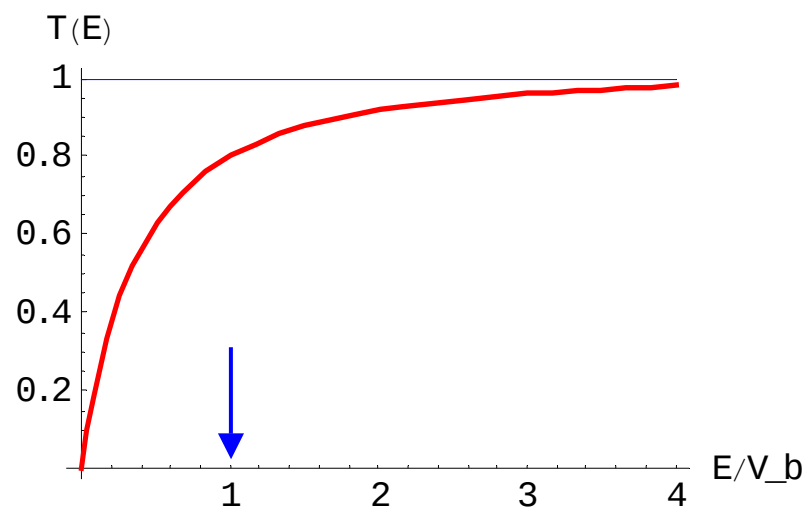
$$k = \sqrt{2mE}/\hbar\ , \quad \kappa = \sqrt{2m(V_b - E)}/\hbar$$

Barrier transmission: Scattering

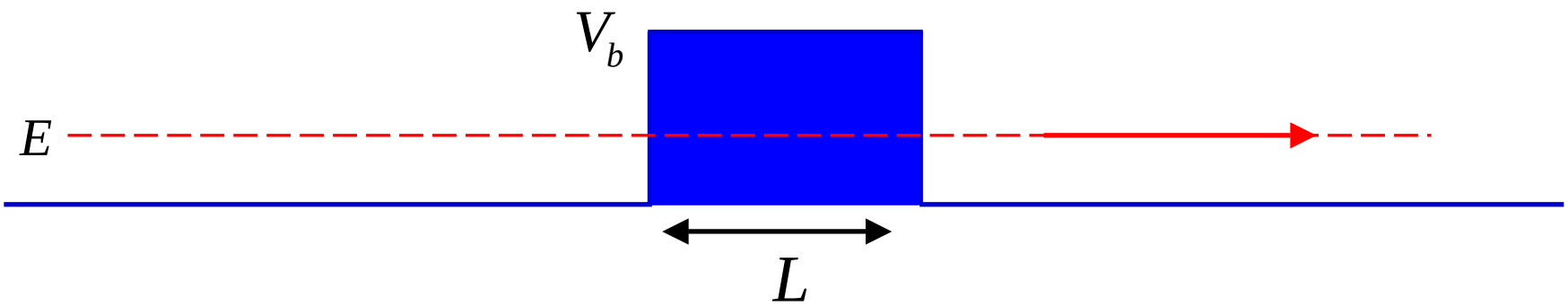


$$T(E) = |t(E)|^2 = \frac{4E(V_b - E)}{4E(V_b - E) + V_b^2 \sinh^2(\kappa L)}$$

$$T(E) + R(E) = 1$$



Barrier transmission: Deep tunneling

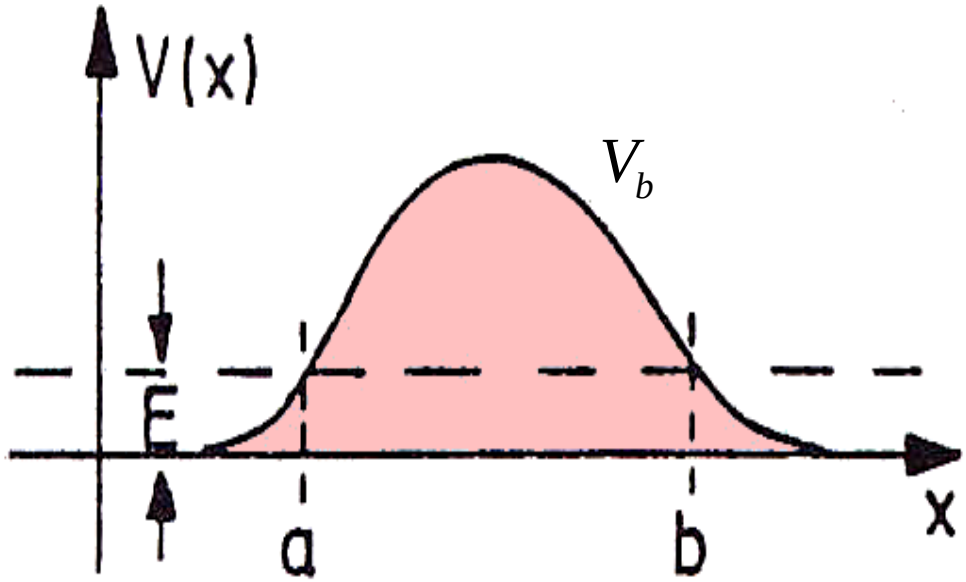


$$V_b \gg E: \quad T(E) \propto \mathrm{e}^{-\underbrace{2\sqrt{2m(V_b-E)}L}/\hbar}$$

$$W(E) = \oint dx \sqrt{-2m(E-V_b)} = \oint dx |p(x)|$$

Imaginary momentum: Euclidian action

Generalization: WKB



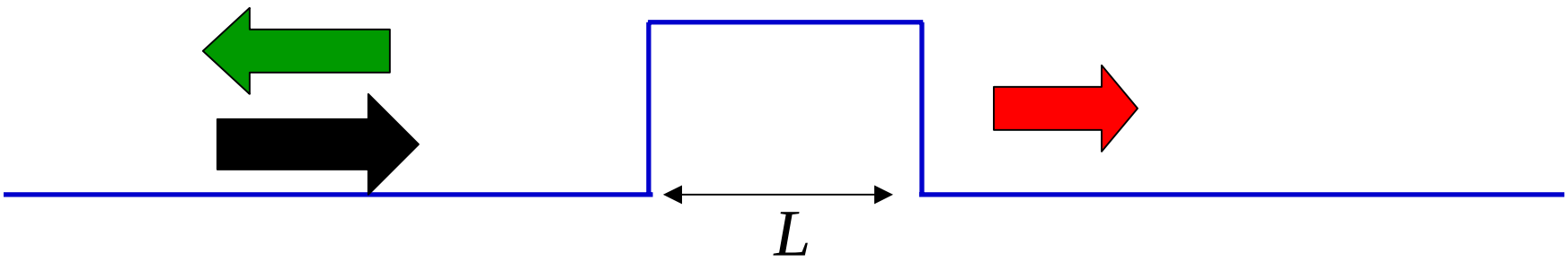
- $\left[\frac{\hat{p}^2}{2m} + V(x) - E\right]\psi(x) = 0 \qquad \psi(x) = \mathrm{e}^{i\tilde{W}(x)/\hbar}\ ,\ \tilde{W} = W_0 - i\hbar W_1 \ldots$

$$W_0(x,a) = \int_a^x dy\ p(y)$$

Smooth and high barrier !
(short wavelengths: geometric optics)

$$T_{\mathrm{WKB}}(E) = \mathrm{e}^{-W(E)/\hbar}\ ,\ W(E) = 2|W_0(b,a)|$$

Incoherent tunneling from a reservoir



Outgoing flux: $j_{\text{out}}(E) = T(E)\, v_{\text{out}}(E)\, \rho(E)\, P(E)$

Density of states per length

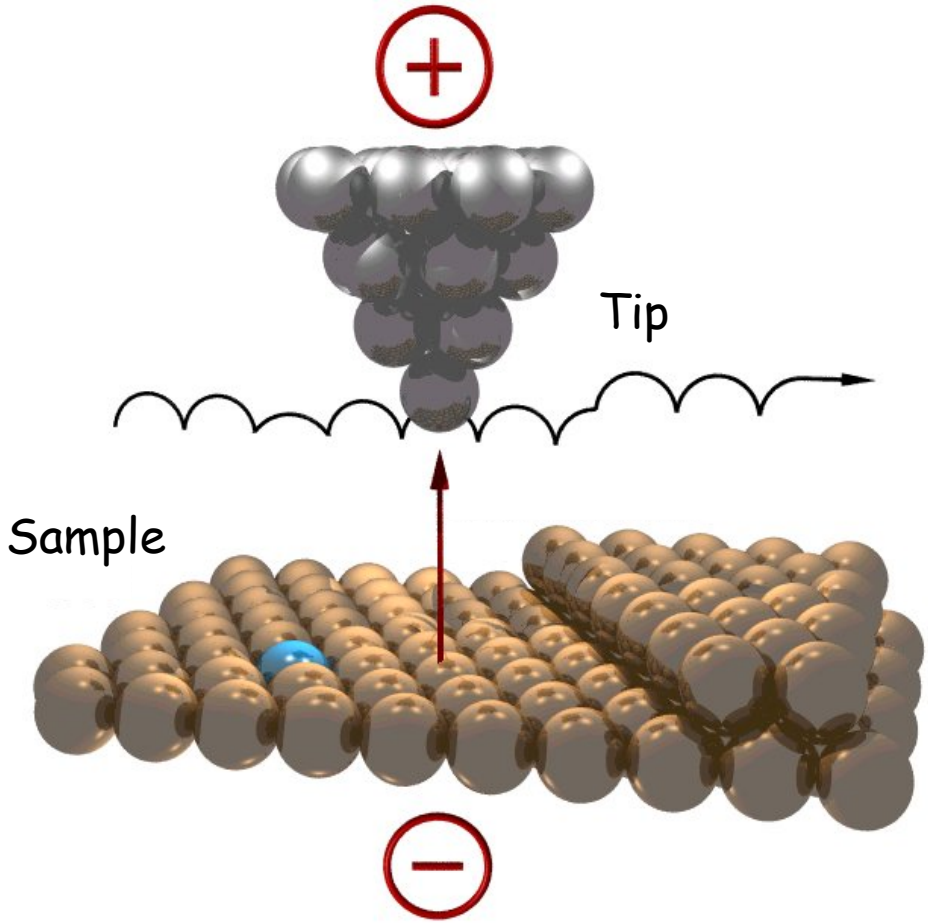
Probability distribution

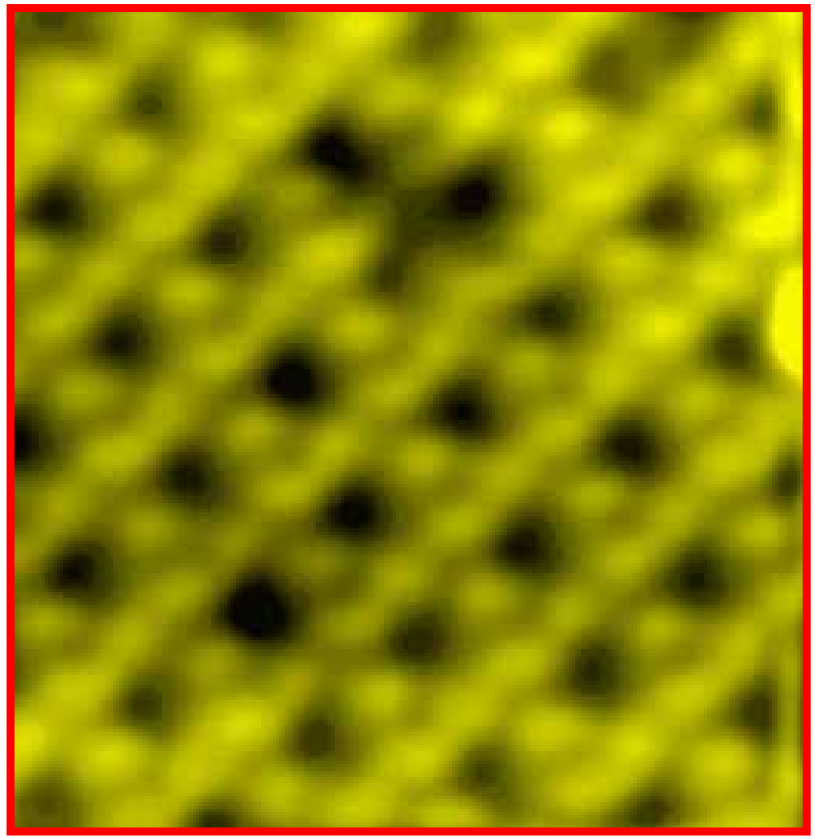
$$v_{\text{out}}(E)\ \rho(E) = \frac{dE}{d\hbar k} \times \frac{dk}{2\pi dE} = \frac{1}{2\pi\hbar}$$

Total rate: $$\Gamma = \frac{1}{Z} \int_0^\infty \frac{dE}{2\pi\hbar}\, T(E)\, P(E)$$

Has dimension of a frequency

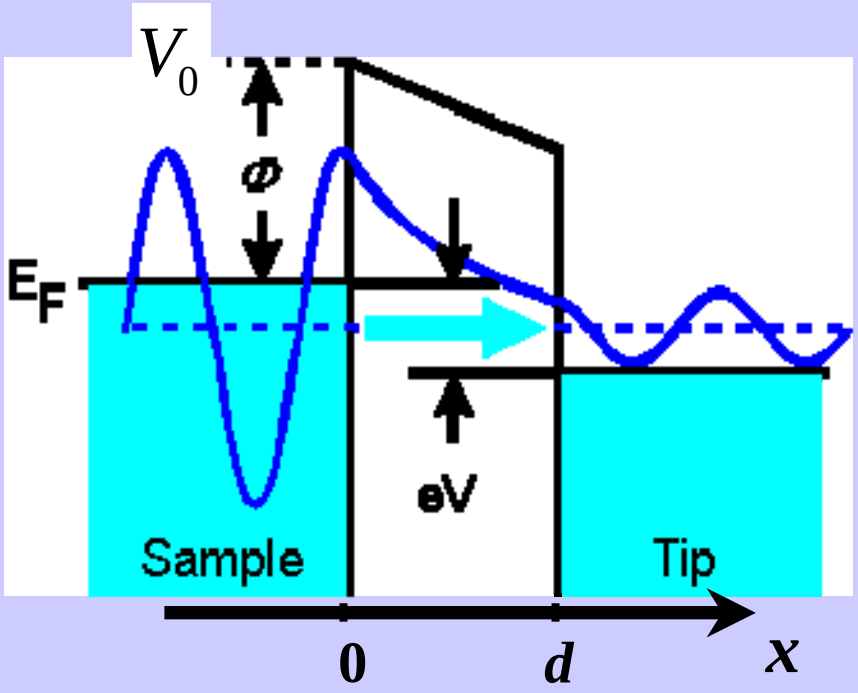
Example:Scanning tunneling microscope





SiC (0001) 3×3 surface

Tunneling resistance



Tunneling current

Temperature = 0:
P(E)=Fermi-function=Step-function

$$I = e \int_{E_F - eV}^{E_F} dE \; T(E) \; \mathrm{v}(E) \rho(E, x=0)$$

Integrand approx. constant near Fermi energy

$$I = \frac{V}{R_T(d)}$$

Tunneling resistance:

$$\frac{1}{R_T(d)} = e\rho(E_F, x=0) \; \mathrm{v}_F \; \mathrm{e}^{-2d\sqrt{2mV_0}/\hbar}$$

Exponential sensitivity

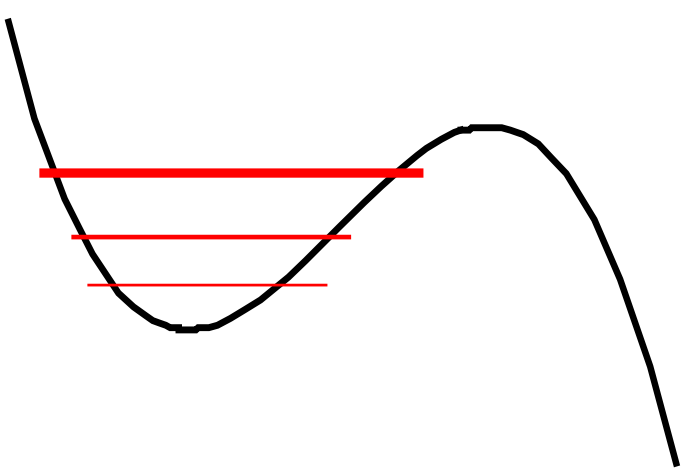
More powerful: ImF

Problem: Energy averaging may be complicated (dissipative systems)

Lecture 2

Idea:

Consider unstable system with quasi-bound states $\epsilon_n = E_n - i\hbar\Gamma_n/2$



$$
\begin{aligned}
Z &= \sum_{n=0}^{\infty} \mathrm{e}^{-\beta\epsilon_n} \\
&\approx \sum_{n=0}^{\infty} \mathrm{e}^{-\beta E_n} + i\frac{\hbar\beta}{2}\sum_{n=0}^{\infty} \Gamma_n \mathrm{e}^{-\beta E_n} \\
&= Z_0\left(1 + i\frac{\hbar\beta}{2}\langle\Gamma_n\rangle\right)
\end{aligned}
$$

with $F = (-1/\beta)\ln Z$

$$\boxed{\Gamma \propto \mathrm{Im}F}$$

More powerful: ImF

Comparison with WKB provides:
I.Affleck, PRL46, 388 (1981)

$$\Gamma = -\phi_T \, \mathrm{Im} F \qquad \phi_T = \frac{2}{\hbar} \begin{cases} T_0/T & \text{for} \quad T \geq T_0 \\ 1 & \text{for} \quad T \leq T_0 \end{cases}$$

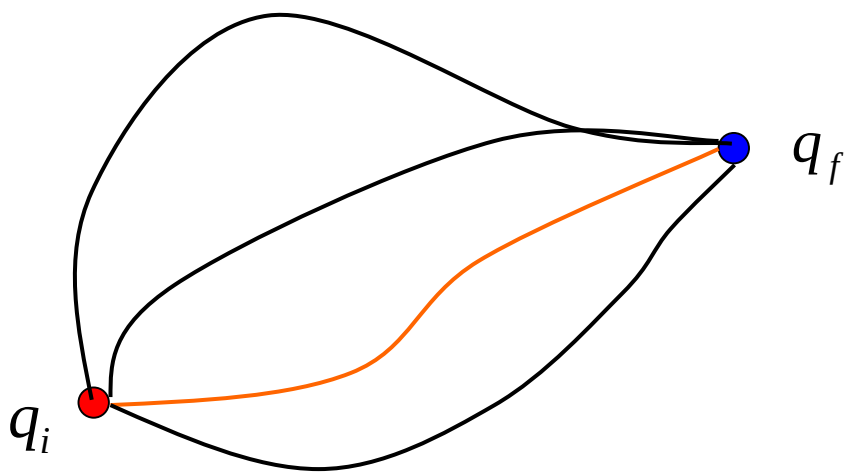
Crossover temperature $T_0 = \dfrac{\hbar \omega_b}{2\pi k_\mathrm{B}}$ (non-dissipative)

How to obtain the free energy ????

Path integrals

Feynman: $$\left\langle q_f \left| \mathrm{e}^{-iHt/\hbar} \right| q_i \right\rangle = \int_{q(0)=q_i}^{q(t)=q_f} D[q]\ \mathrm{e}^{iS[q]/\hbar}$$

"Sum over all paths"



Path integrals in imaginary time

$$\mathrm{e}^{-iHt/\hbar} \xrightarrow{t \to -i\hbar\beta} \mathrm{e}^{-\beta H}$$

Path integrals in imaginary time

Partition function: $$Z = \mathrm{Tr}\{\mathrm{e}^{-\beta H}\} = \oint \mathcal{D}[q]\ \mathrm{e}^{-S_E[q]/\hbar}$$

$$S_E[q] = \int_0^{\hbar\beta} d\tau \left[\frac{m\dot{q}^2}{2} + V(q)\right]$$

Sum over all periodic paths in $\tau \in [0, \hbar\beta]$

running in the inverted potential

How to evaluate ?

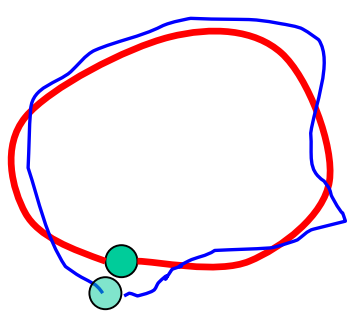
For sufficiently high barriers:

Integral dominated by least action paths

(but remember that we are interested in „ImF")

Least action paths: $m\ddot{q} - V'(q) = 0 \ , \quad q(0) = q(\hbar\beta)$

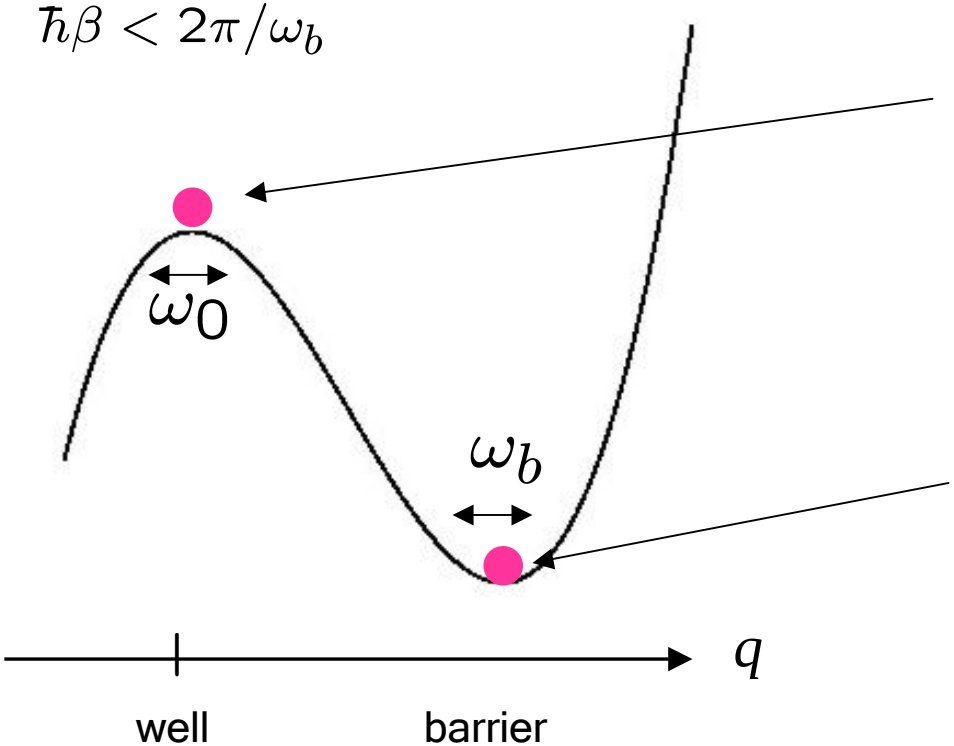
Fluctuations: $$Z \approx \sum_{\alpha} D_\alpha \ \mathrm{e}^{-S[q_\alpha]/\hbar}$$



Up to second order: Gaussian integrals

Metastable well: high temperature

$\hbar\beta < 2\pi/\omega_b$



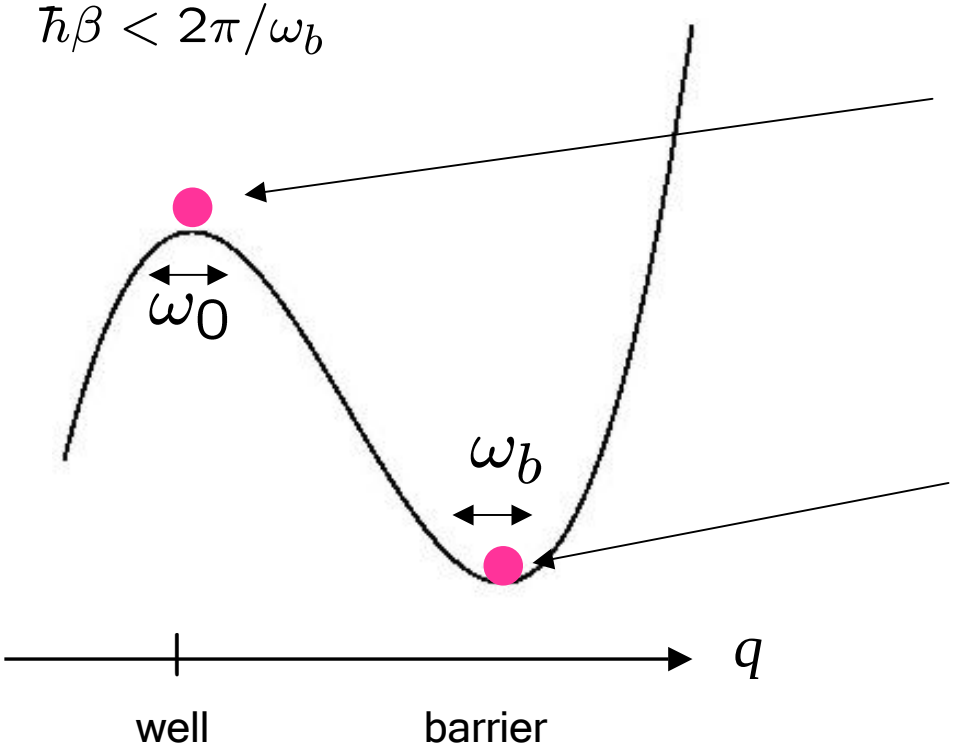
Stable: $S[q = q_0] = 0$

Unstable: $S[q = q_b] = \beta V_b$

Metastable well: high temperature

Thermal activation + quantum fluctuations

$\hbar\beta < 2\pi/\omega_b$



Stable: $S[q = q_0] = 0$

Unstable: $S[q = q_b] = \beta V_b$

Gaussian fluctuations
In analytic continuation

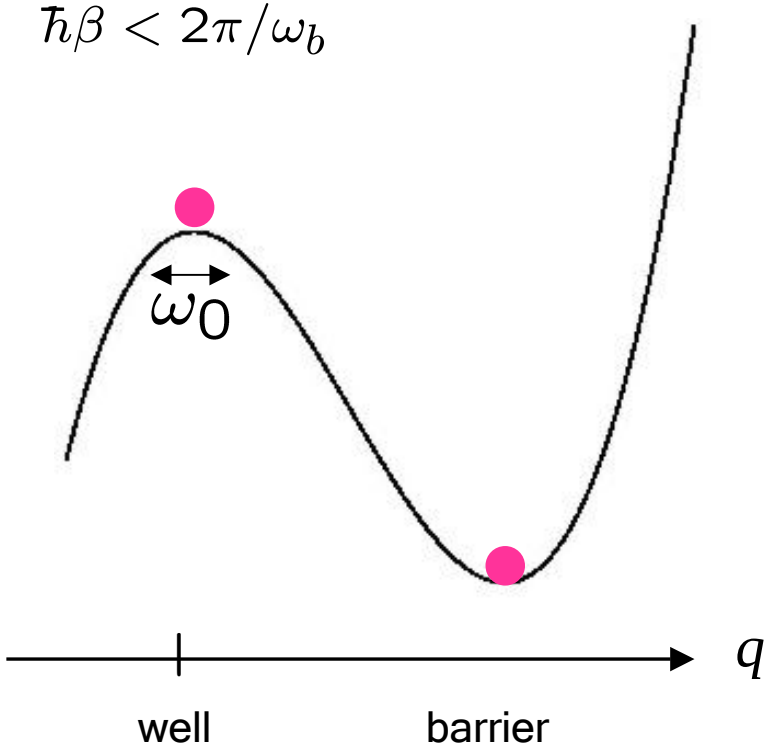
$$\Gamma = \frac{\omega_0}{2\pi} \frac{\sinh(\omega_0 \hbar\beta/2)}{\sin(\omega_b \hbar\beta/2)} e^{-\beta V_b}$$

Imaginary part

Metastable well: Low temperature

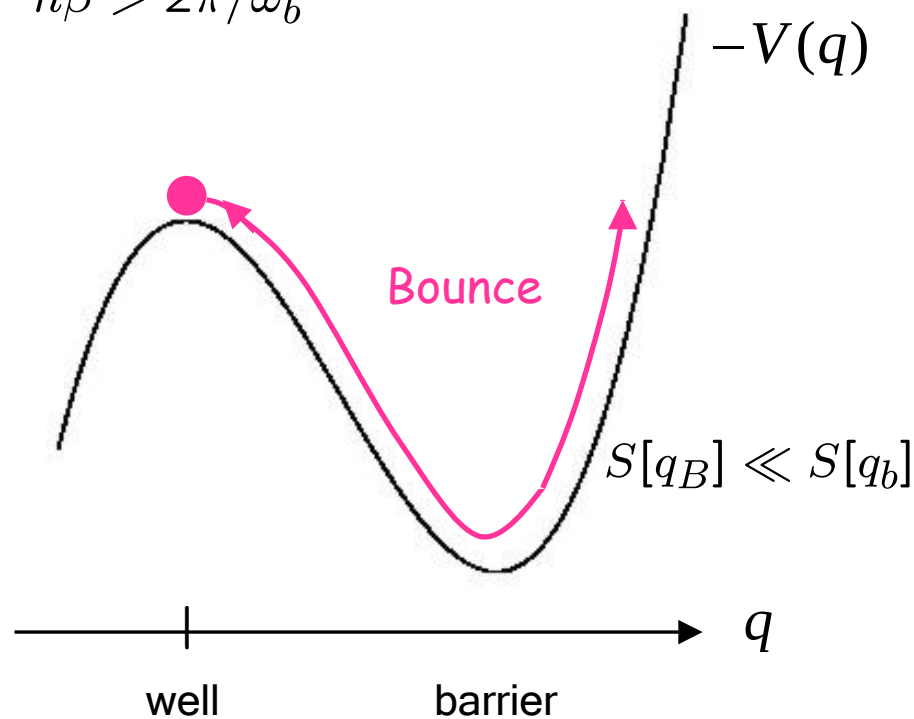
Quantum tunneling

$\hbar\beta < 2\pi/\omega_b$



$$\Gamma = \frac{\omega_0}{2\pi} f_q \, \mathrm{e}^{-\beta V_b}$$

$\hbar\beta > 2\pi/\omega_b$



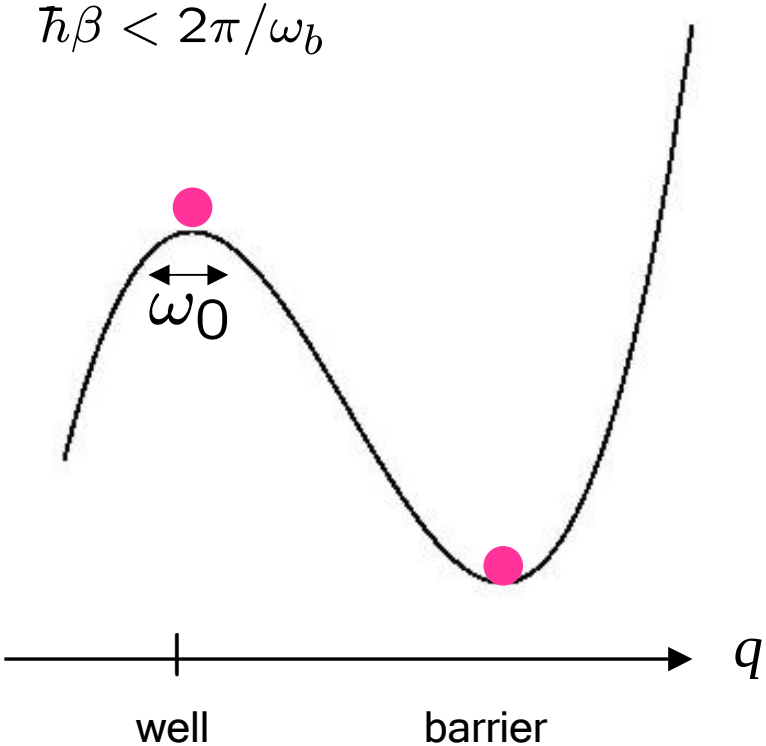
$$\Gamma = \frac{\omega_0}{2\pi} F_q \; \mathrm{e}^{-S[q_B]/\hbar}$$

cubic: $S_B = \frac{36}{5}\frac{V_b}{\omega_0}$

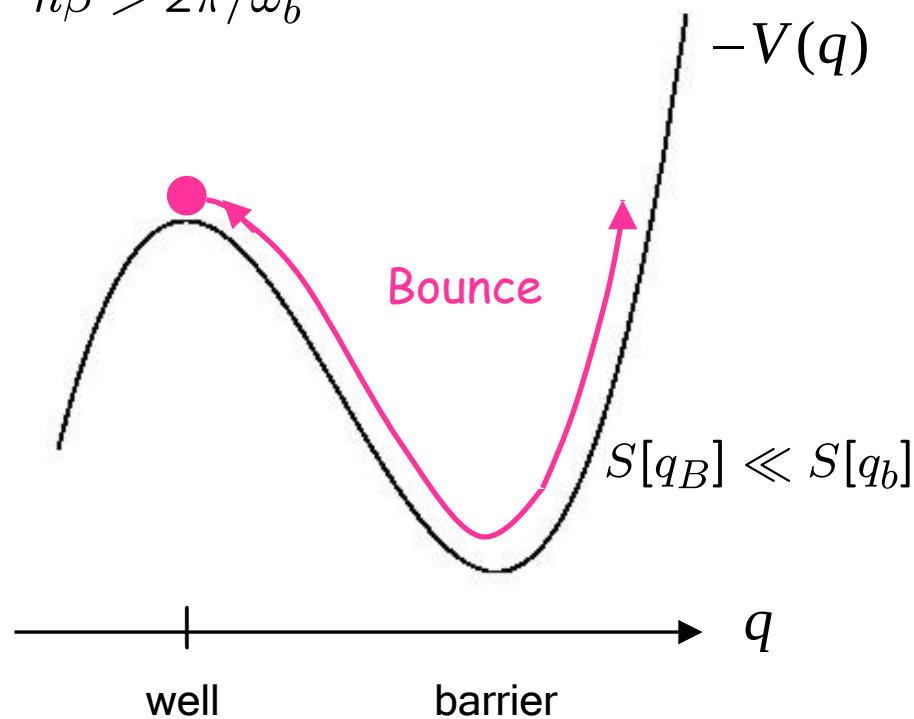
Metastable well: Low temperature

Quantum tunneling

$\hbar\beta < 2\pi/\omega_b$



$\hbar\beta > 2\pi/\omega_b$



$$\Gamma = \frac{\omega_0}{2\pi} f_q \, \mathrm{e}^{-\beta V_b}$$

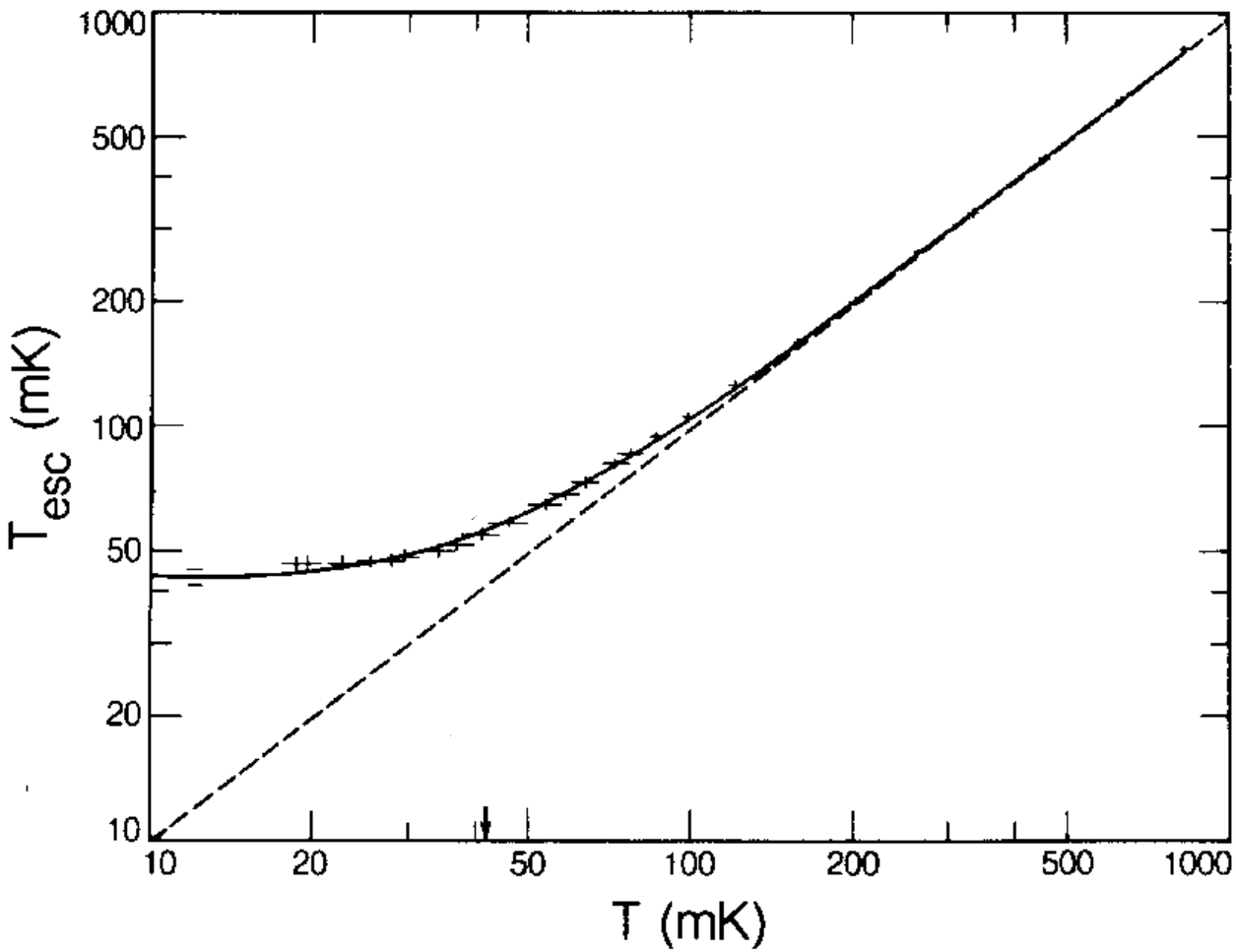
$$\Gamma = \frac{\omega_0}{2\pi} F_q \, \mathrm{e}^{-S[q_B]/\hbar}$$

Crossover $\omega_b \hbar\beta = 2\pi$

cubic: $S_B = \frac{36}{5}\frac{V_b}{\omega_0}$

Crossover

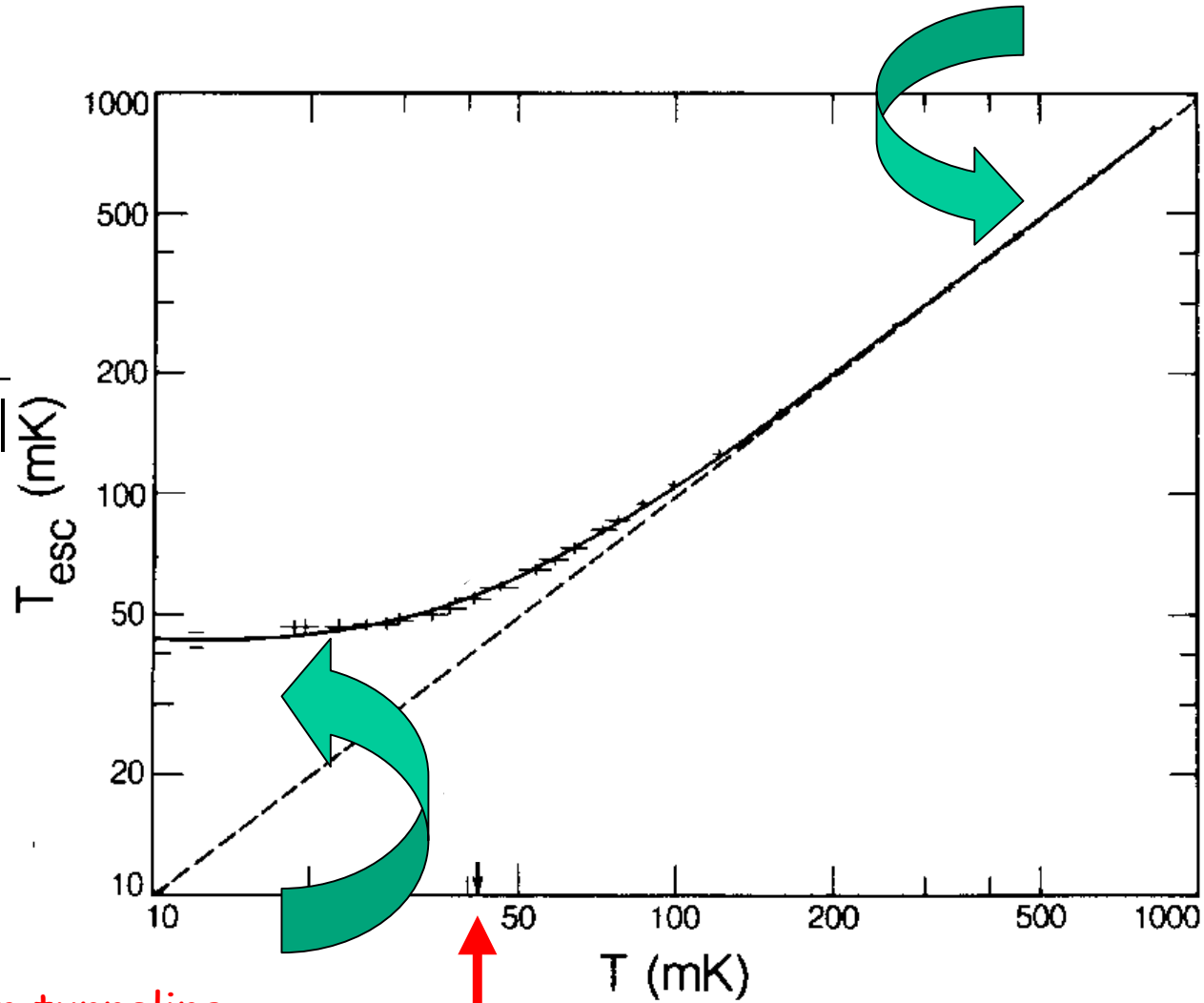
$$\frac{const}{|\mathrm{Ln}(\Gamma)|}$$



Crossover

Thermal activation

$$\frac{const}{|\mathrm{Ln}(\Gamma)|}$$

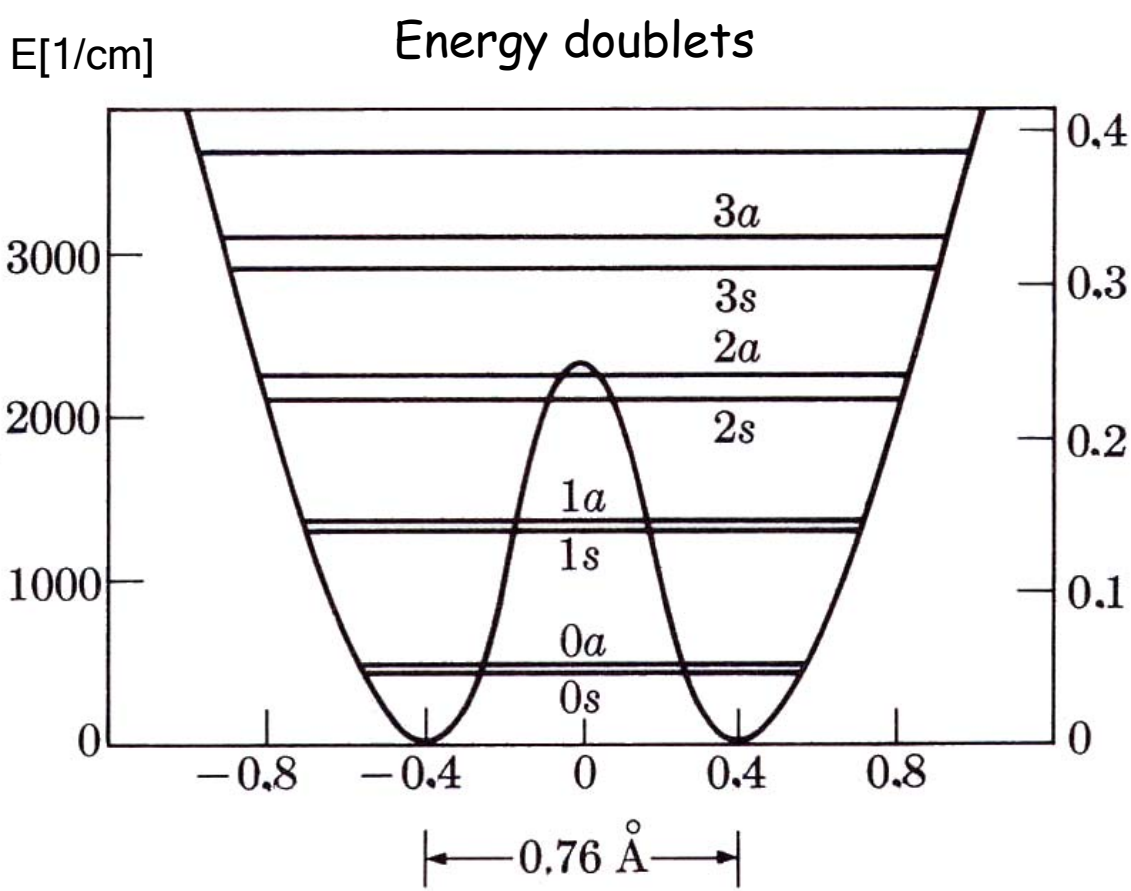


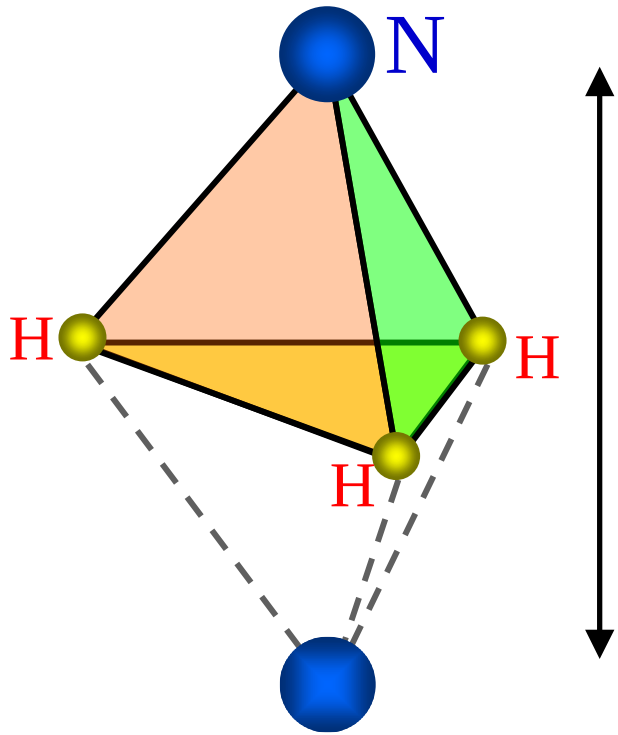
Quantum tunneling

T_0

Coherent tunneling

Double well


E[1/cm]
Energy doublets
3000
2000
1000
0
0.4
0.3
0.2
0.1
0
3a
3s
2a
2s
1a
1s
0a
0s
−0.8
−0.4
0
0.4
0.8
0.76 Å


N
H
H
H

Coherent tunneling

Ground state

Localized basis: $|0L\rangle$ $|0R\rangle$

2-state Hamiltonian: $H = \begin{pmatrix} \hbar\omega/2 & \hbar\Delta \\ \hbar\Delta & \hbar\omega/2 \end{pmatrix}$

Coherent tunneling

Ground state doublet

Localized basis: $|0L\rangle$, $|0R\rangle$

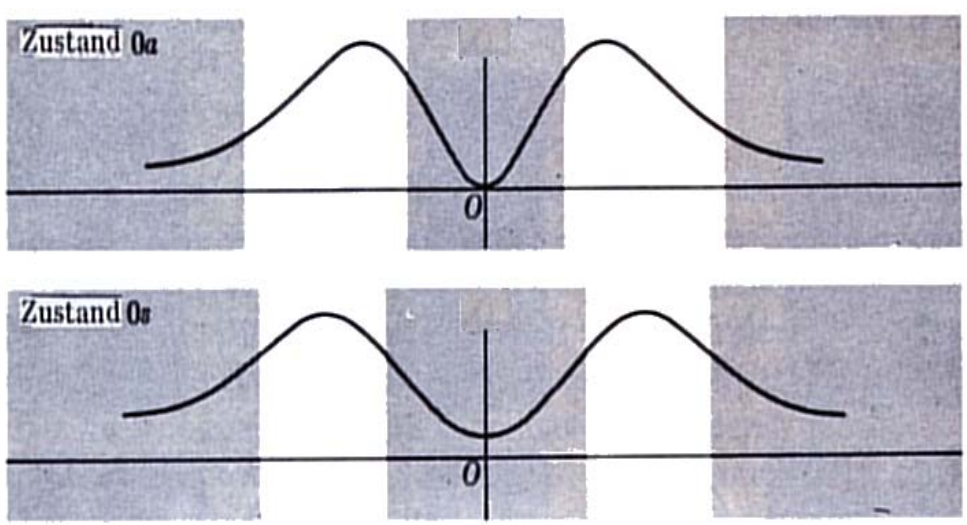
2-state Hamiltonian: $H = \begin{pmatrix} \hbar\omega/2 & \hbar\Delta \\ \hbar\Delta & \hbar\omega/2 \end{pmatrix}$

Eigenbasis:

$$|0a\rangle = \frac{1}{\sqrt{2}}\left(|0L\rangle - |0R\rangle\right)$$

$$|0s\rangle = \frac{1}{\sqrt{2}}\left(|0L\rangle + |0R\rangle\right)$$

Energies: $E_{\pm} = \frac{\hbar\omega}{2} \mp \hbar\Delta$



Coherent tunneling dynamics

Initial state:

$$|\psi(0)\rangle = (|0s\rangle - |0a\rangle)/\sqrt{2} \equiv |0R\rangle$$

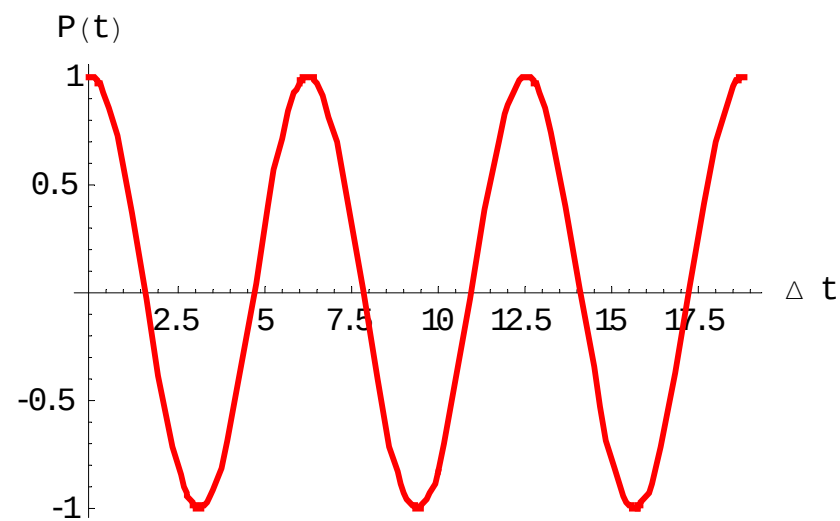
Time evolution:

$$|\psi(t)\rangle| = \mathrm{e}^{-i\omega t/2}\ (\mathrm{e}^{i\Delta t}|0s\rangle - \mathrm{e}^{-i\Delta t}|0a\rangle/\sqrt{2}$$

Overlap:

$$\langle\psi(0)|\psi(t)\rangle = \cos(\Delta t)$$

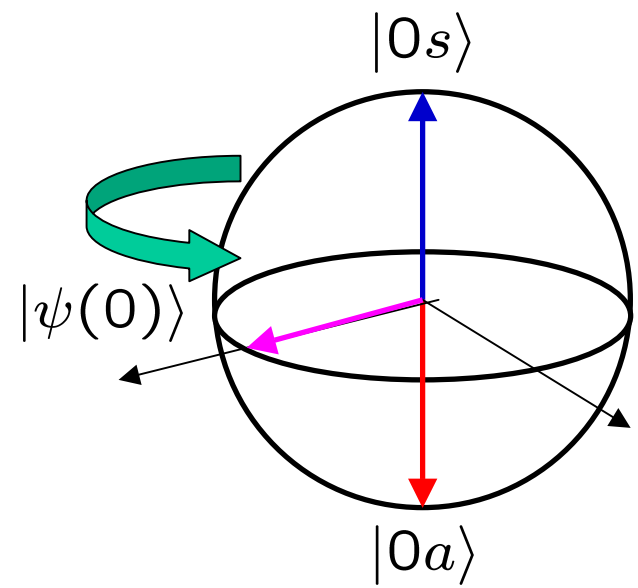
Spectroscopy measures Δ



Bloch sphere

Initial state:

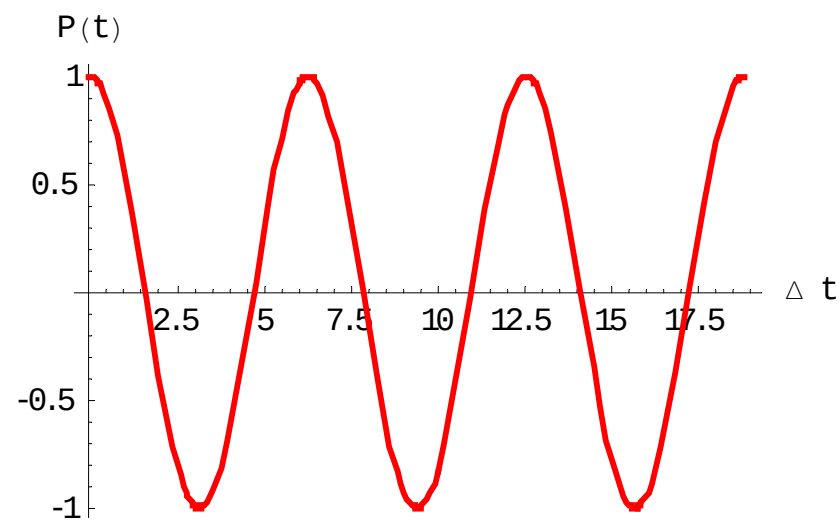
$$|\psi(0)\rangle = (|0s\rangle - |0a\rangle)/\sqrt{2} \equiv |0R\rangle$$



In eigenstate basis:

$$H = \frac{\hbar\omega}{2} I + \hbar\Delta\sigma_z$$

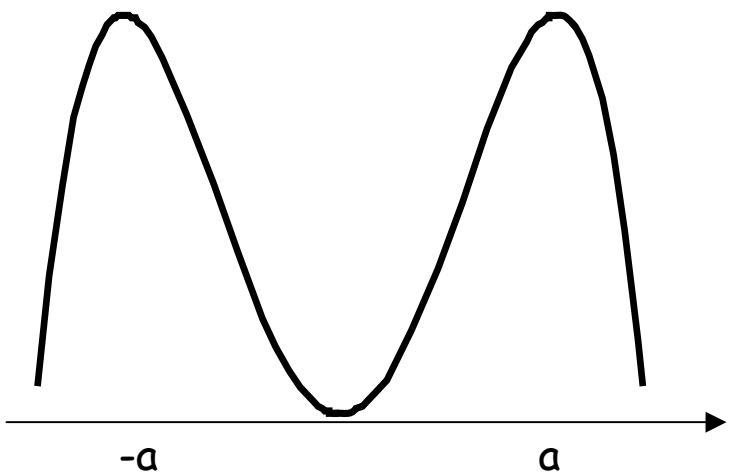
Particle in a magnetic field in z-direction

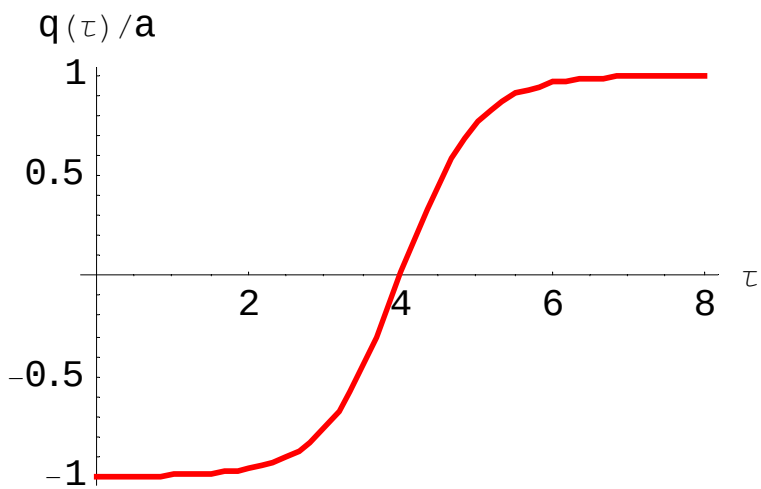


Tunnel splitting from partition function

$$Z = \mathrm{Tr}\, e^{-\beta H} = \oint \mathcal{D}[q]\; e^{-S_E[q]/\hbar}$$

Least action paths in inverted double well for very low T:



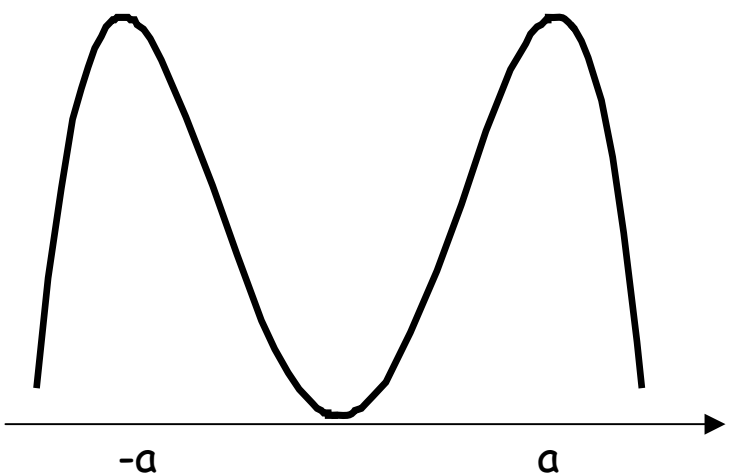


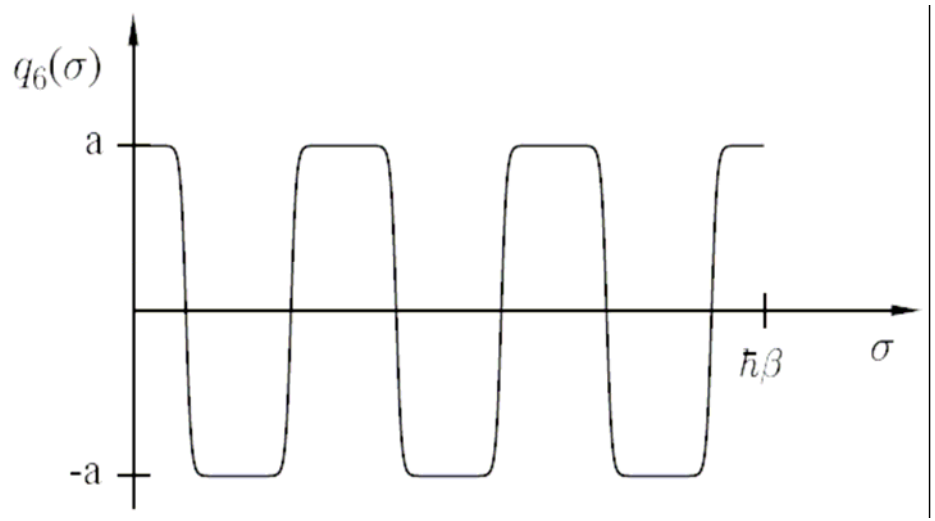
Instanton

Tunnel splitting from partition function

$$Z = \mathrm{Tr}\, e^{-\beta H} = \oint \mathcal{D}[q]\ e^{-S_E[q]/\hbar}$$

$\hbar\beta$ very large: Family of paths with n instanton/anti-instanton pairs, n=0,1,2,...





n=3

Tunnel splitting from partition function

$$Z = \mathrm{Tr}\, e^{-\beta H} = \oint \mathcal{D}[q]\ e^{-S_E[q]/\hbar}$$

Sum over all pairs and their fluctuations in a time ordered manner:
(zero mode!)

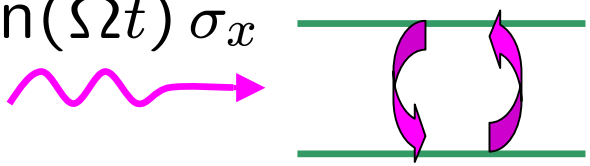
$$Z = e^{-\beta\hbar\omega/2}\ \left(e^{\beta\hbar\Delta} + e^{-\beta\hbar\Delta}\right)$$

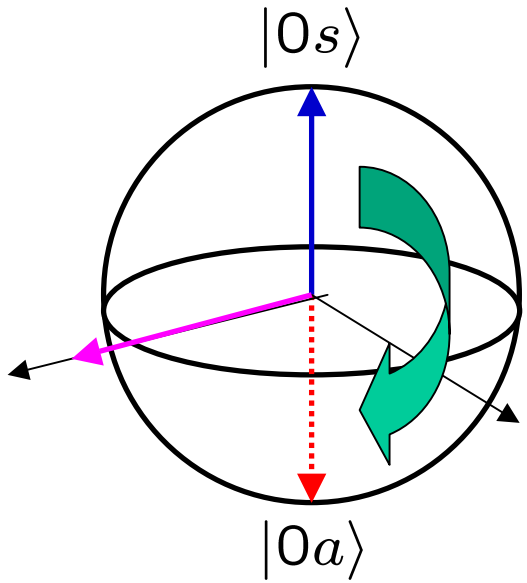
$$\Delta \propto \omega_0\, e^{-W(E=0)/2\hbar}$$

Instanton action

External driving: Rabi oscillations

$A \sin(\Omega t)\, \sigma_x$





Microwave field in resonance with energy splitting:

oscillations with frequency $\propto A$

Summary Lecture 1:

- Coherent and incoherent tunneling
- Transmissions and averaged transmissions (decay rates)
- Partition function: Euclidian path integral (inverted pot.)
  - incoherent decay: ImF, bounce orbit (unstable)
  - coherent tunneling: instanton

Thanks

Set-up

Lecture 1: Basics and tools

- Introduction
- Simple examples
- WKB-approximation for transmission probabilities
- Decay rates and ImF-technique
- Quantum Coherence

Lecture 2: Tunneling in Macroscopic Systems ?

- MQC and MQT ?
- Dissipative tunneling
- Nanomagnets: coherent and incoherent tunneling
- SQUIDS
- BEC

Goal: Understanding MQT and MQC

So far: Tunneling of microscopic objects

Is tunneling observable for systems with macroscopically many degrees of freedom ?

Is quantum mechanics valid for these systems ?

And if we assume yes: How should we describe them ?

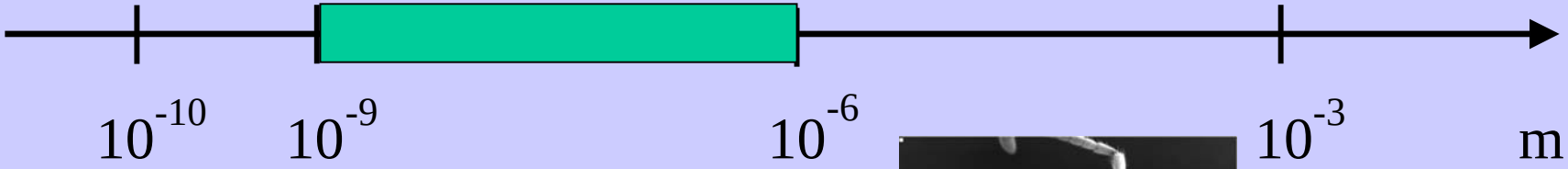
Why is tunneling not observable in our world ?

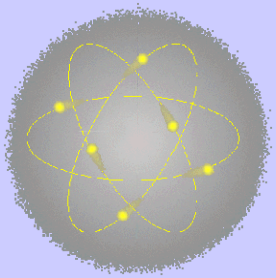
MQT, MQC: Tunneling of collective degrees of freedom

These systems are embedded in a surrounding

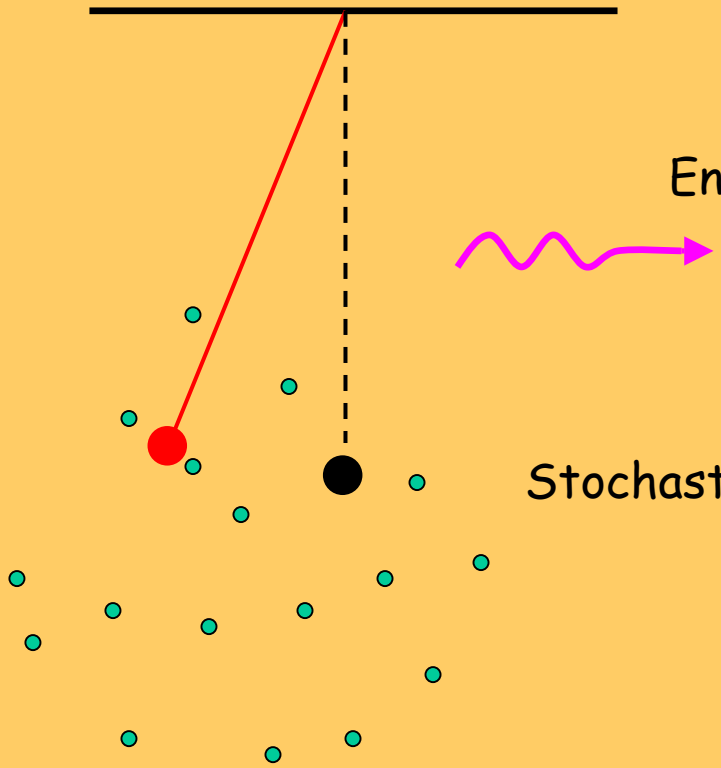
they interact with the macroscopic world

Open systems: energy exchange
feel fluctuating forces





Classical: Open systems



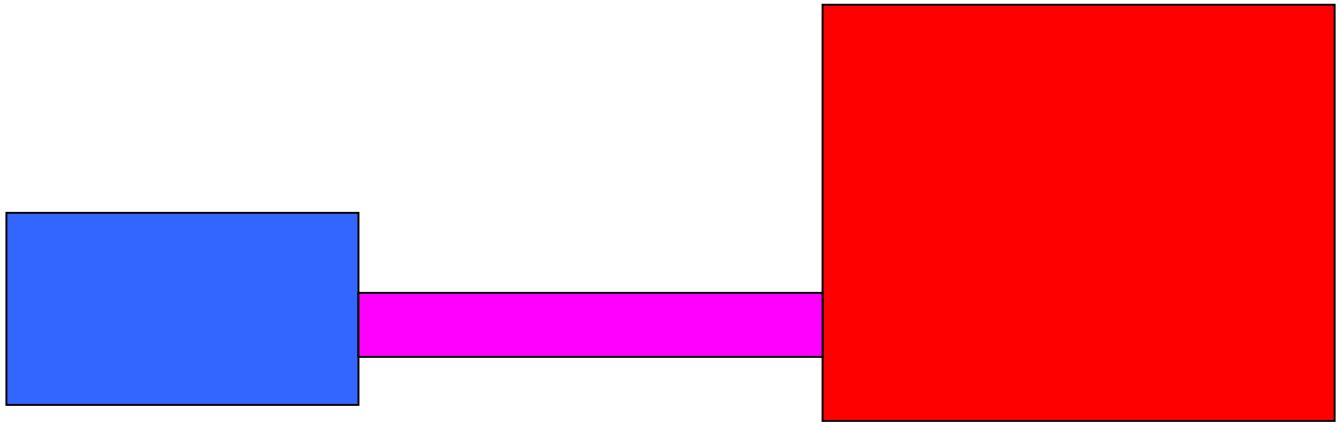
Energy flow: Relaxation

Coupling constant

Stochastic scattering: Fluctuations

Temperature

Open quantum systems



$$H_S \quad + \quad H_I \quad + \quad H_R$$

System + reservoir: reduced density

$$\rho(t) = \mathrm{Tr}_{\mathrm{R}} \left\{ \mathrm{e}^{-iHt/\hbar} W(0)\, \mathrm{e}^{iHt/\hbar} \right\} \longrightarrow \rho_\beta = \frac{1}{Z} \mathrm{Tr}_{\mathrm{R}} \left\{ \mathrm{e}^{-\beta H} \right\}$$

Caldeira-Leggett-Model:

(Magalinskii [1959], Ullersma [1966], Zwanzig [1973])

$$
\begin{aligned}
H &= H_S + H_I + H_B \\
H_I + H_B &= \sum_{i=1}^{\infty} \left[\frac{p_i^2}{2m_i} + \frac{m_i \omega_i^2}{2} \left(x_i - \frac{c_i}{m_i \omega_i^2} q \right)^2 \right]
\end{aligned}
$$

Harmonic oscillator bath with continuum of modes, thermal distribution

Classical: generalized Langevin equation for system

$$
M\ddot{q} + M \int_0^{\infty} ds \; \gamma(t-s)\dot{q}(s) + V'(q) = \xi(t)
$$

Friction kernel

Noise

Caldeira-Leggett-Model:

(Magalinskii [1959], Ullersma [1966], Zwanzig [1973])

Friction determined by spectral bath density:

$$\gamma(t) = \frac{2}{M}\int_0^\infty \frac{d\omega}{\pi}\frac{I(\omega)}{\omega}\cos(\omega t)$$

Noise determined by dissipation-fluctuation theorem:

$$\langle \xi(t)\xi(s)\rangle = M k_{\mathrm{B}} T \gamma(t-s)$$

Markovian friction (ohmic, Johnson-Nyquist):

$$\langle \xi(t)\xi(s)\rangle = 2 M k_{\mathrm{B}} T \gamma(t-s)$$

Influence of the bath given by

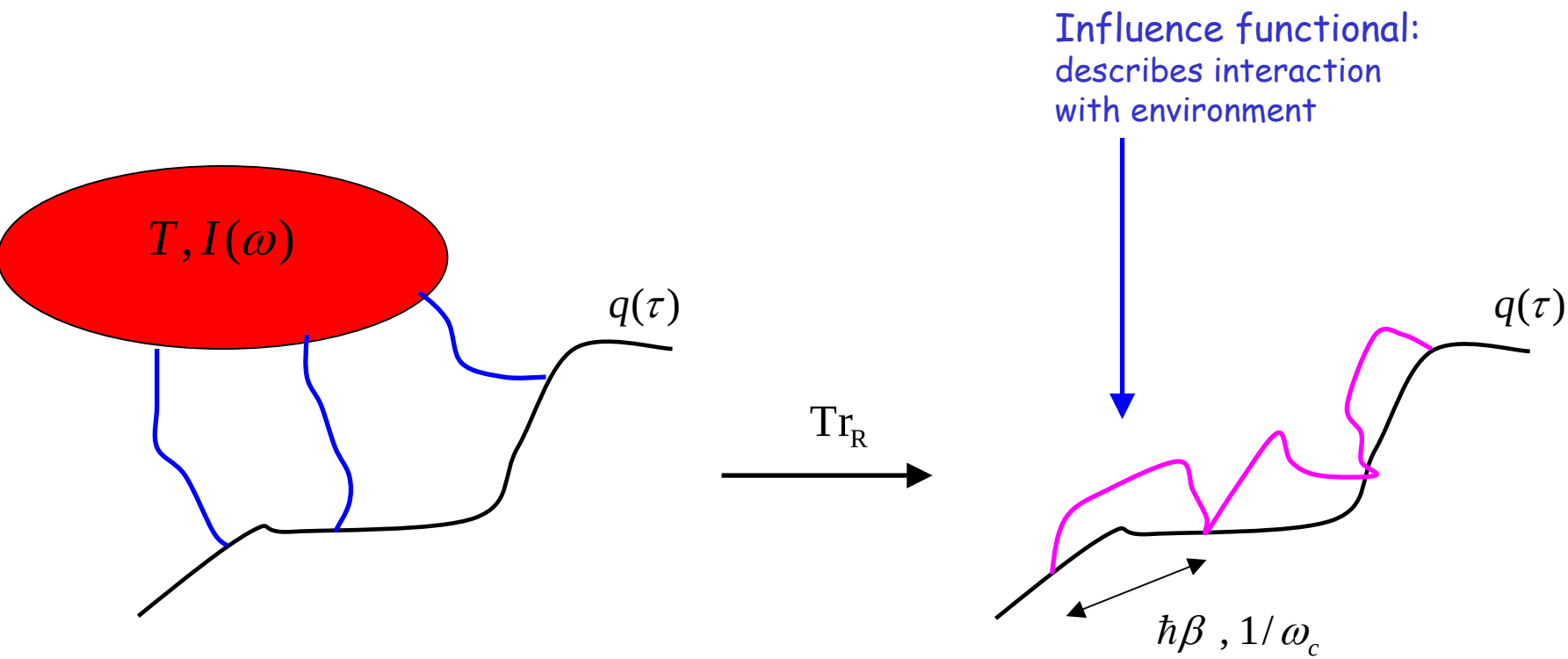
$$T \ , \ I(\omega)$$

Gaussian statistics of the bath fluctuations

Quantization: Path integrals

Path integral in imaginary time after carrying out the Gaussian integrals:

$$Z = \oint \mathcal{D}[q]\ \mathrm{e}^{-S_E[q]/\hbar}\ \mathrm{e}^{-\Phi[q]/\hbar}$$

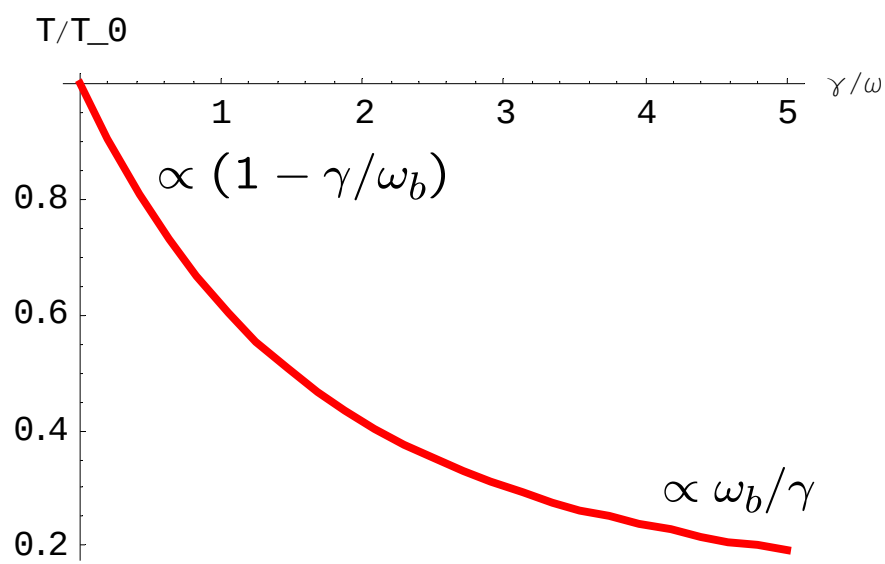


Macroscopic quantum tunneling

Crossover: $T_0 = \frac{\hbar\omega_\gamma}{k_B 2\pi}$

$$\omega_\gamma^2 + \omega_\gamma \tilde{\gamma}(\omega_\gamma) - \omega_b^2 = 0 \ , \ \omega_\gamma > 0$$

Fourier transform of $\gamma(t)$



Ohmic friction: $\tilde{\gamma}(\omega) = \gamma$

Fricition suppresses quantum effects

Macroscopic quantum tunneling rates

Above crossover

$$\Gamma = \frac{\omega_0}{2\pi}\frac{\omega_\gamma}{\omega_b}\left[\prod_{n=1}^{\infty}\frac{\nu_n^2+\omega_0^2+\nu_n\tilde{\gamma}(\nu_n)}{\nu_n^2-\omega_b^2+\nu_n\tilde{\gamma}(\nu_n)}\right]\ e^{-\beta V_b}$$

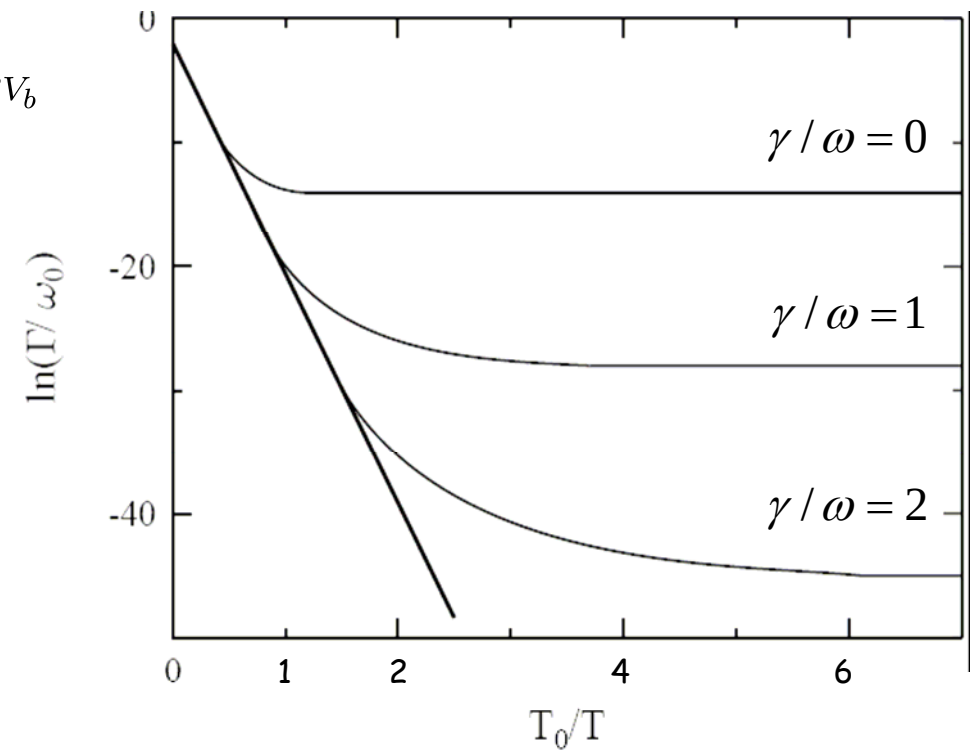
Below crossover

$$\Gamma = \sqrt{\frac{S_B}{2\pi\hbar}}\left[\prod_{n=1}^{\infty}\frac{\lambda_n^{(0)}}{\lambda_n^{(b)}}\right]\ e^{-S_B/\hbar}$$

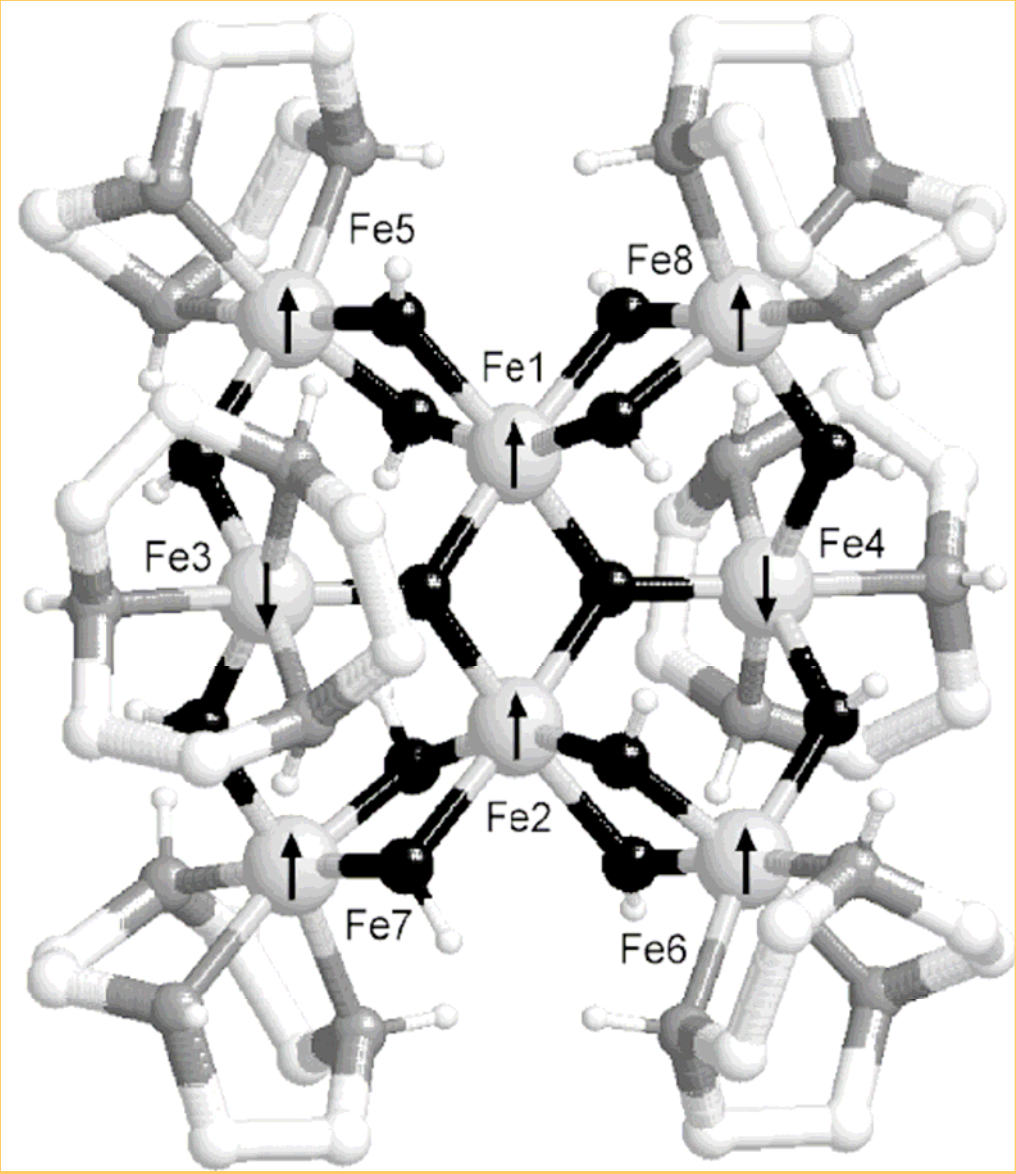
$$\gamma \text{ small}: \quad S_B \approx \frac{36}{5}\frac{V_b}{\omega_0}[1+O(\gamma)]$$

$$\gamma \text{ large}: \quad S_B \approx 3\pi\gamma V_b/\omega_0^2$$

Rate suppression

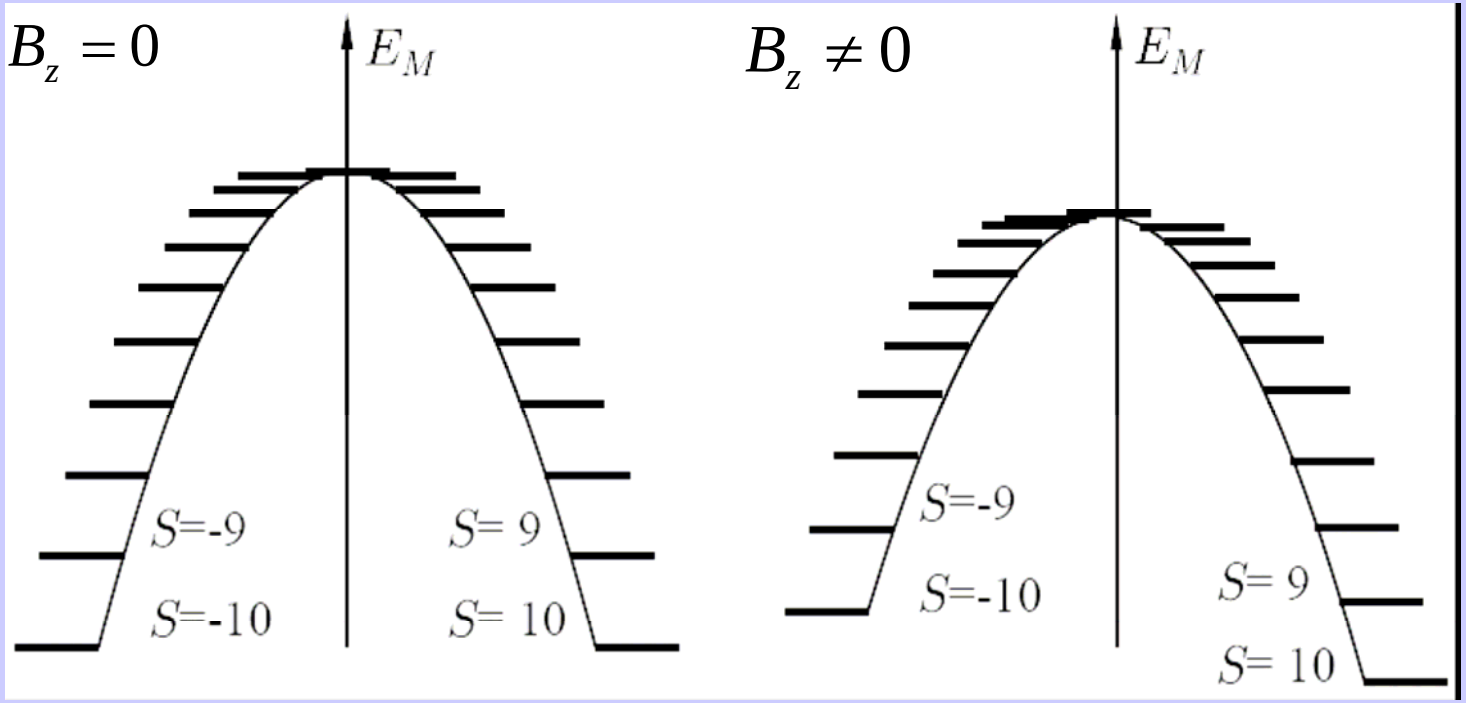


Example: Tunneling of magnetization



Wernsdorfer

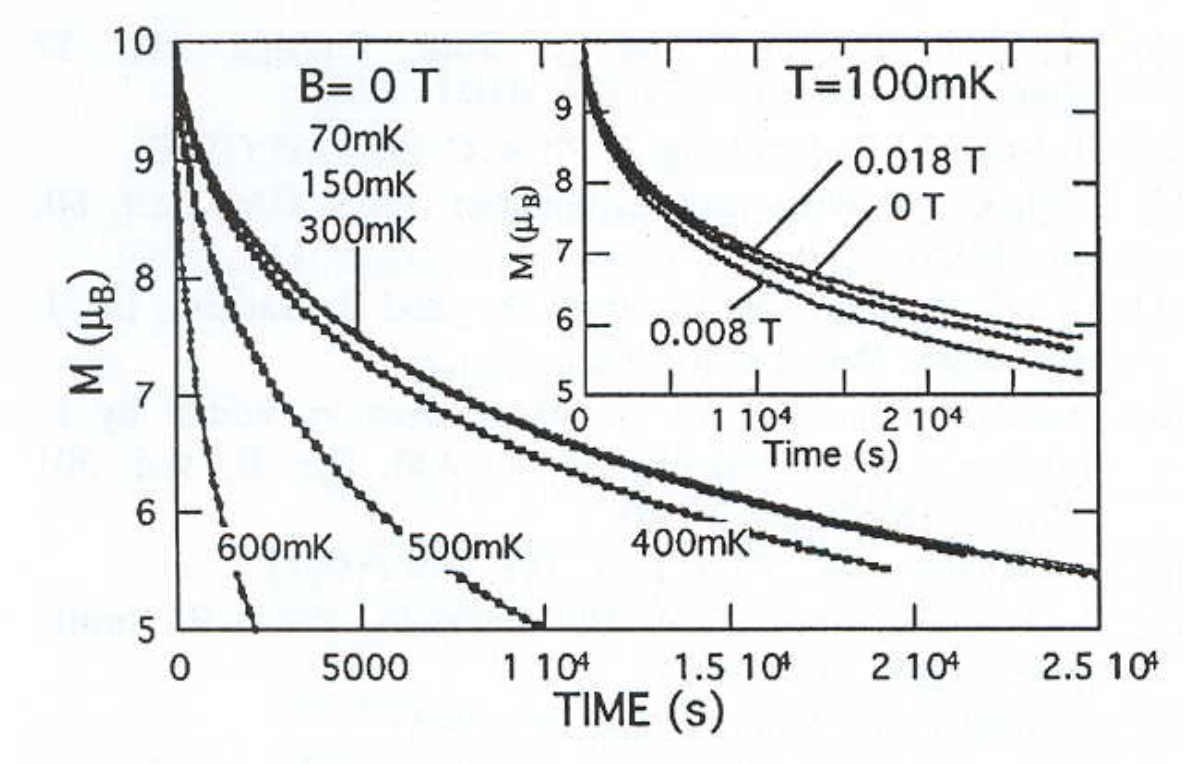
Tunneling of magnetization in Fe_8 clusters



$$H = -DS_z^2 + E(S_x^2 - S_y^2) - gS_z B_z$$

Anisotropy: induces tunneling

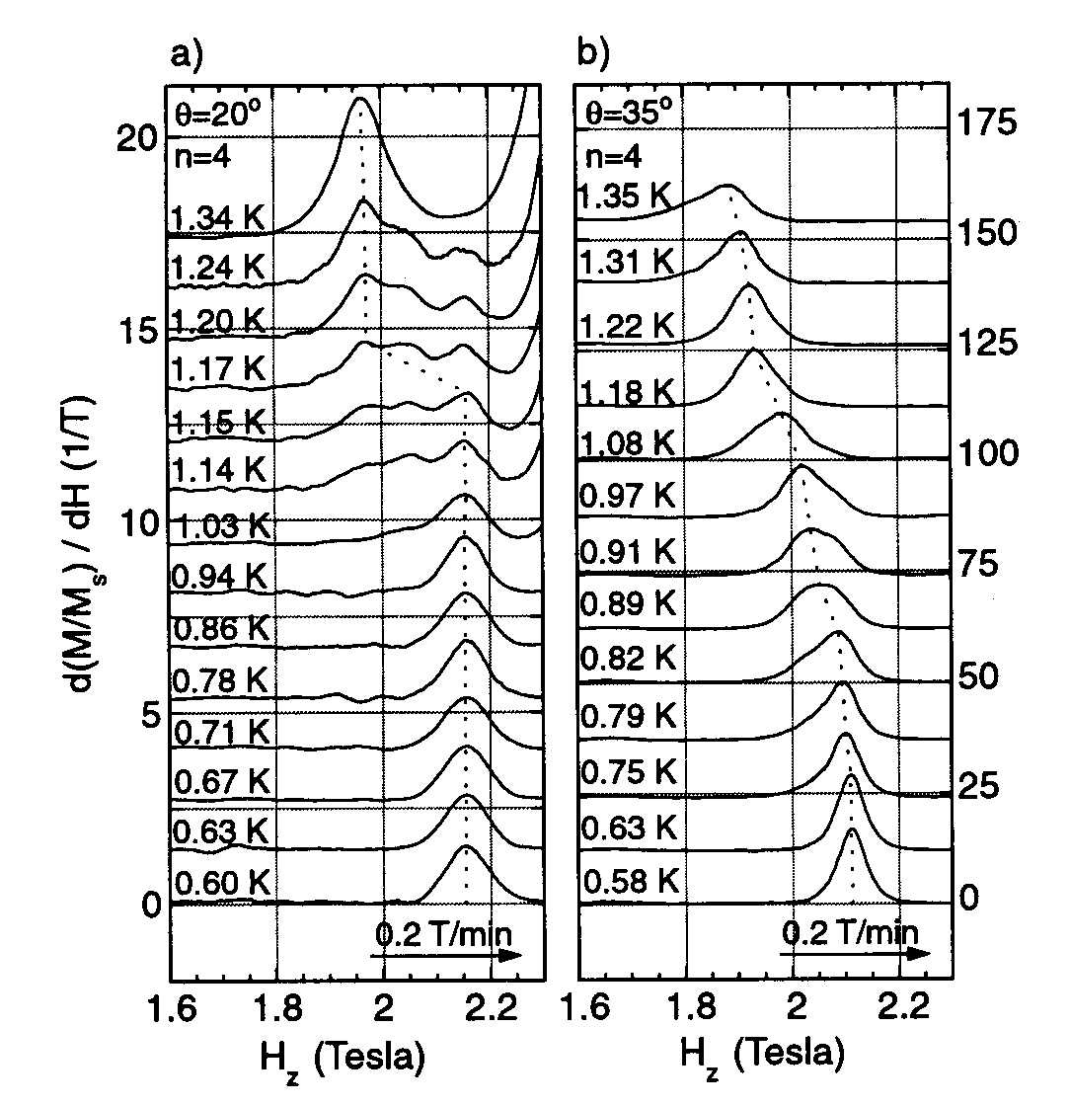
MQT in iron clusters



Sangregorio et al, PRL 78, 4645 (1997)

Influence of the environment: phonon induced transitions

Mn_{12}



Bokacheva et al, PRL 85, 4803 (2000)

Reduced real-time dynamics: Master equations

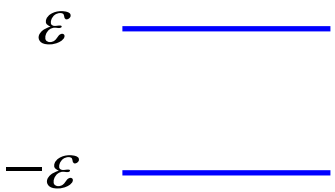
Master/Redfield-Equation

2. order perturbation theory in H_I

$$i\hbar \frac{d\rho(t)}{dt} = [H_S, \rho(t)] + \gamma^2 \mathcal{R}[\rho(t)]$$

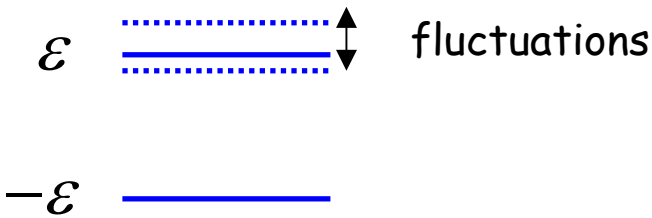
- Various types: CL-Master equation, Lindblad, ...
- numerically efficient
- weak friction: $\gamma \hbar \beta \ll 1$
- fast bath modes: $\omega_0 \ll \omega_c$

Dephasing and relaxation



$$H = \begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}$$

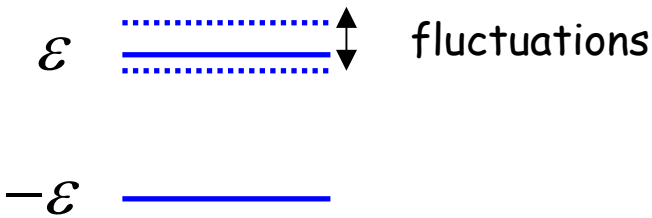
Dephasing and relaxation



$$H = \begin{pmatrix} \epsilon + \xi(t) & 0 \\ 0 & -\epsilon - \xi(t) \end{pmatrix}$$

Dephasing: Noise couples to the eigenstates
Affects only the non-diagonal elements of the density

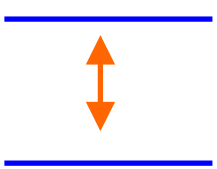
Dephasing and relaxation



$$H = \begin{pmatrix} \epsilon + \xi(t) & \Delta \\ \Delta & -\epsilon - \xi(t) \end{pmatrix}$$

Dephasing: Noise couples to the eigenstates
Affects only the non-diagonal elements of the density

Relaxation: transitions, leads to equilibrium
affects also the diagonal elements



$e^{-\beta E_n}$

Thermal bath

Statistics of the bath coupling operator $X = \sum_{i=1}^{\infty} c_i x_i$

Spectral power:

$$S(\omega) = \frac{1}{2}\langle\{X(t), X(s)\}\rangle_\omega = \hbar\, I(\omega)\, \coth(\beta\hbar\omega/2)$$

$$\langle\sigma_\pm(t)\rangle \to \langle\sigma_\pm(0)\rangle\, \mathrm{e}^{\mp i2\epsilon t/\hbar}\ \mathrm{e}^{-\gamma_{\mathrm{dep}} t}$$

$$\langle\sigma_z(t)\rangle = \sigma_z^\infty + [\langle\sigma_z(0)\rangle - \sigma_z^\infty]\, \mathrm{e}^{-\gamma_{\mathrm{rel}} t}$$

$$\gamma_{\mathrm{dep}} = \frac{\gamma_{\mathrm{rel}}}{2} + \eta^2\ S(\omega = 0)$$

Dephasing rate

$$\gamma_{\mathrm{rel}} = \eta^2\ S(\omega = 2\epsilon)$$

Relaxation rate

Thermal bath

Statistics of the bath coupling operator $X = \sum_{i=1}^{\infty} c_i x_i$

Spectral power:

$$S(\omega) = \frac{1}{2}\langle\{X(t), X(s)\}\rangle_\omega = \hbar\, I(\omega)\, \coth(\beta\hbar\omega/2)$$

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$$\langle\sigma_z(t)\rangle = \sigma_z^\infty + [\langle\sigma_z(0)\rangle - \sigma_z^\infty]\, \mathrm{e}^{-\gamma_{\mathrm{rel}} t}$$

$$\gamma_{\mathrm{dep}} = \frac{\gamma_{\mathrm{rel}}}{2} + \eta^2\ S(\omega = 0)$$

$\to 0$ for $T \to 0$

$$\gamma_{\mathrm{rel}} = \eta^2\ S(\omega = 2\epsilon)$$

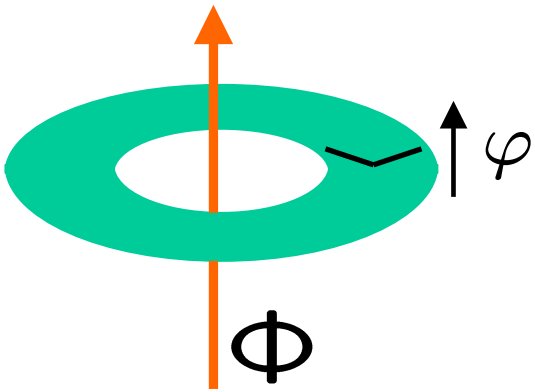
Examples

Examples

Examples

Example 1: Supercondcuting Quantum Interference Device

Rf-SQUID



$$\varphi = -2\pi(\Phi/\varphi_0 - n) \quad , \quad \Phi = \Phi_x + L\, I_S$$

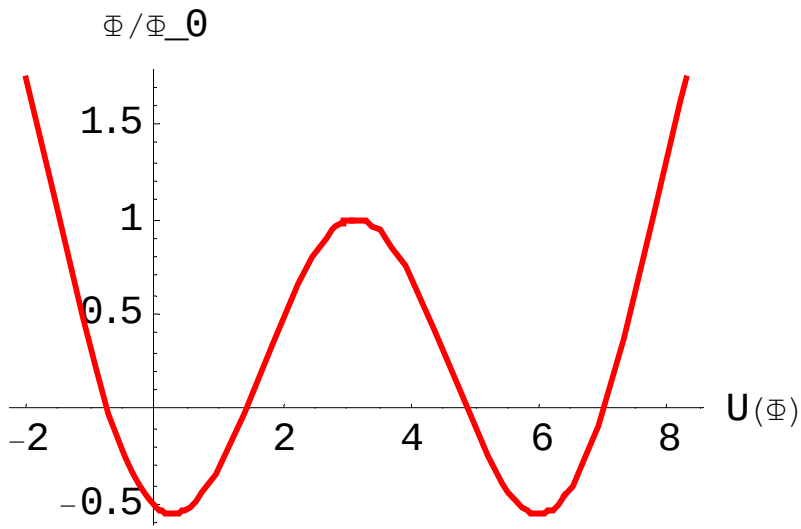
$$\varphi_0 = \hbar/2e$$

Dynamical variable is now Φ

Potential:

$$U(\Phi) = \frac{(\Phi - \Phi_x)^2}{2L} - E_J \cos(\Phi/\varphi_0)$$

Lecture 3

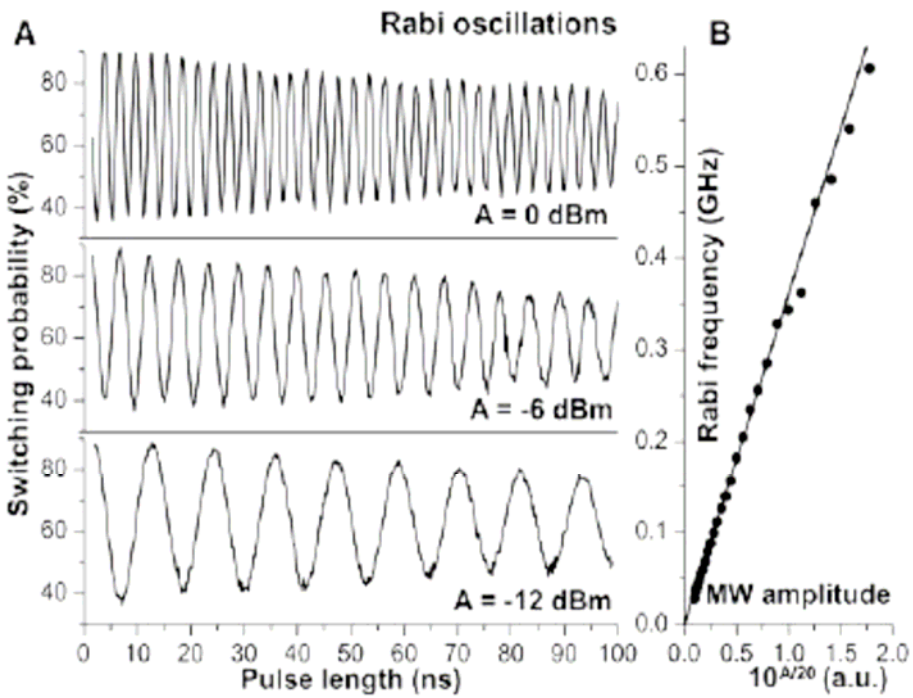


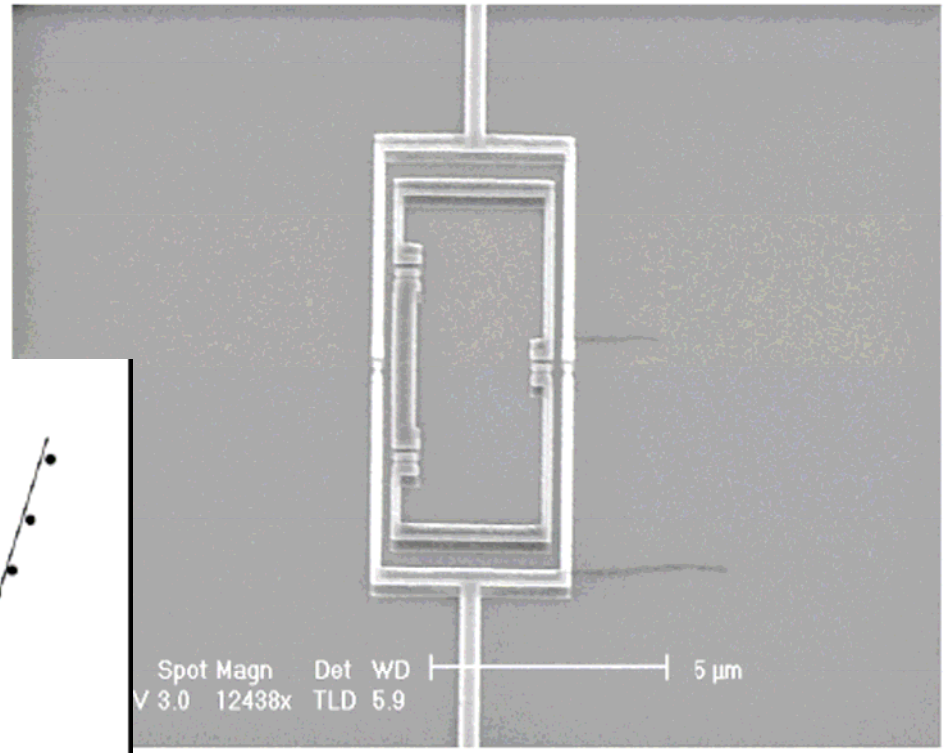
$$\beta_L = LE_J/\varphi_0^2 = 10, \;\; \Phi_x = \pi\varphi_0$$

DC-SQUID

Less sensitive to noise

Rabi oscillations for
the ground state doublet

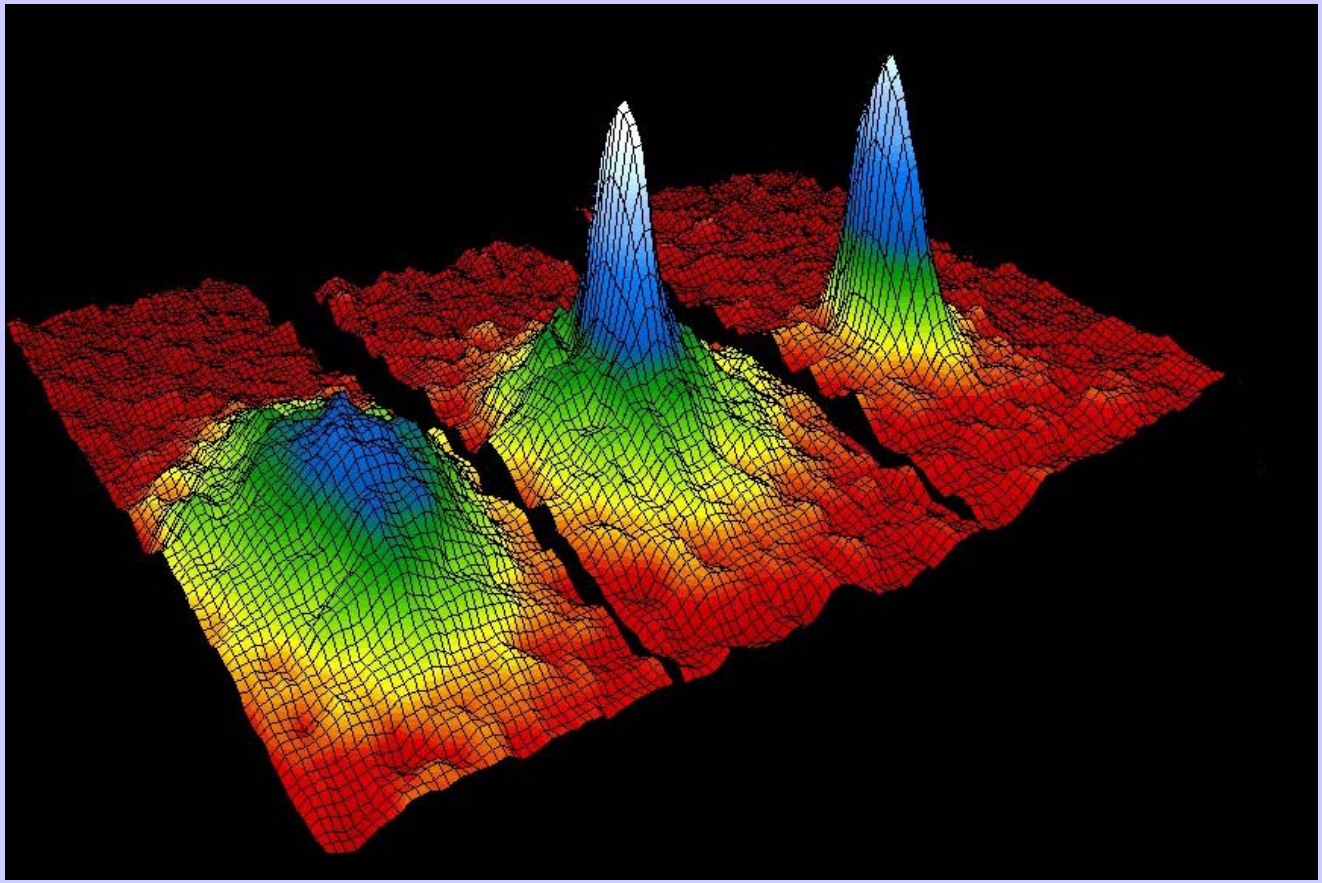




A. Lupascu, ENS

Chiorescu et al, Science 229, 1869 (2003).

Example 2: **B**ose **E**instein **C**ondensate



Wineland et al

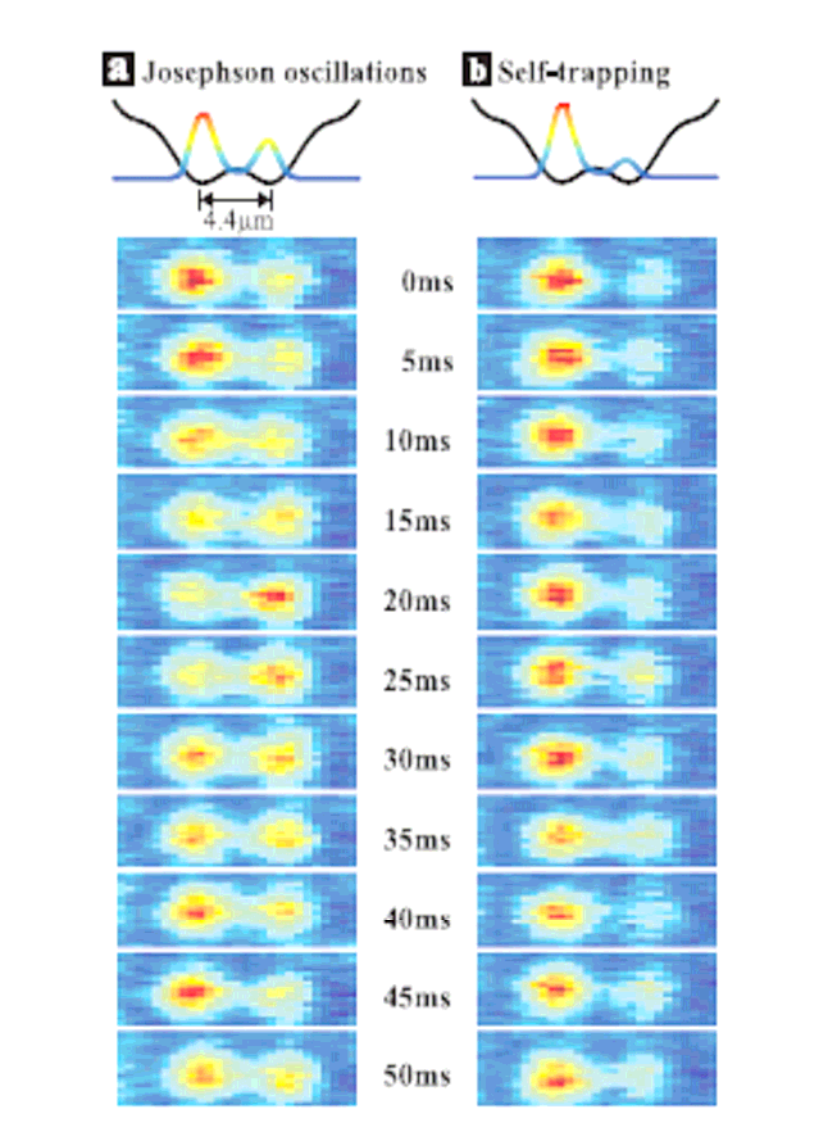
Coherent Tunneling of a BEC

$$z = (N_l - N_r)/N \ , \ \phi = \phi_r - \phi_l$$

Two-mode approximation:

$$\begin{aligned} \dot{z} &= -\sqrt{1-z^2}\,\sin\phi \\ \dot{\phi} &= \Lambda z + \frac{z}{\sqrt{1-z^2}}\cos\phi \end{aligned}$$

Non-rigid pendulum with angular momentum z



M. Albiez et al, PRL 95, 010402 (2005)

$z(0) < z_c$ $\quad$ $z(0) > z_c$

Summary Lecture 2:

- MQT and MQC: tunneling of collective dof
- embedded in environment: dissipative tunneling
- Nanomagnet, SQUID, BEC

....vortices, Helium-3,4...

Thanks

Set-up

Lecture 1: Basics and Theoretical Tools

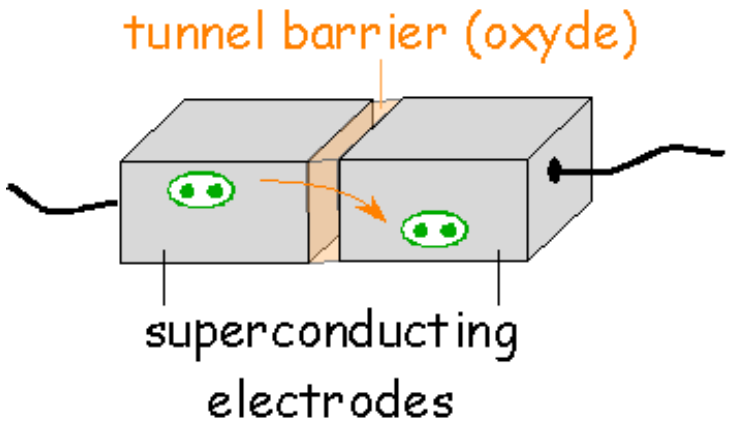
Lecture 2: Tunneling in Macroscopic Systems ?

Lecture 3: Tunneling in Josephson systems

- Josephson junctions
- Switching rates
- From classical activation to MQT
- Experimental results
- Tunneling of quantum bits

Goal: Understanding MQT and MQC in Josephson devices

Josephson-junction



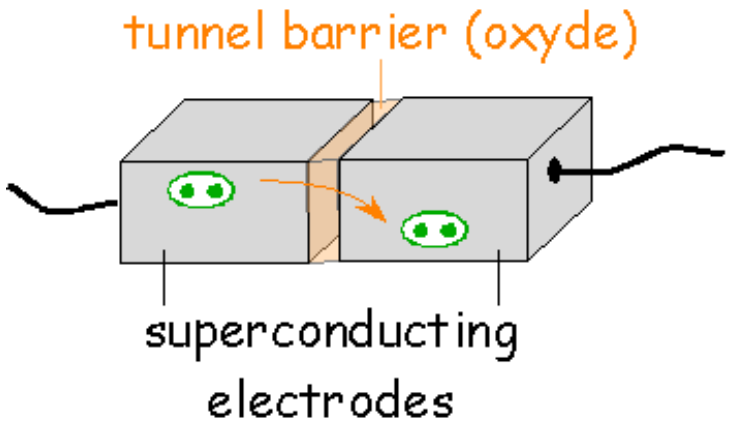
$$\langle\psi|L\rangle = \sqrt{\rho}\ \mathrm{e}^{i\varphi_L} \qquad \langle\psi|R\rangle = \sqrt{\rho}\ \mathrm{e}^{i\varphi_R}$$

Two wave functions overlap !

$$H = E_L\,|L\rangle\langle L| + E_R\,|R\rangle\langle R| + E_J\,(|L\rangle\langle R| + |R\rangle\langle L|)$$

$$E_R - E_L = eV$$

Josephson-junction



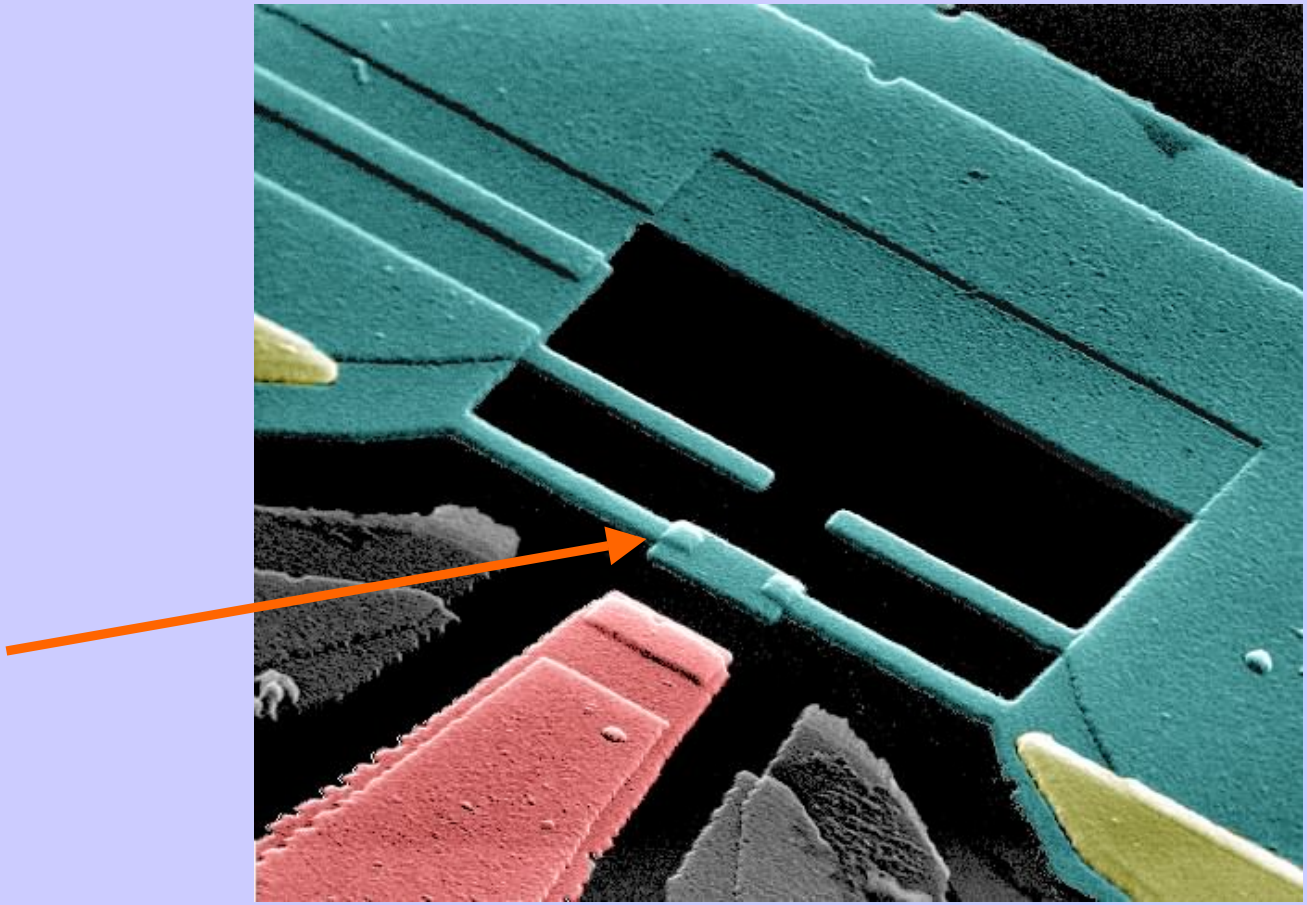
Solve time dependent Schrödinger equation for ρ , $\varphi = \varphi_R - \varphi_L$

Josephson relations:

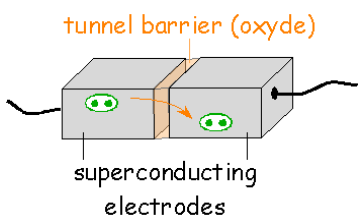
$$I = I_0 \sin(\varphi) \ , \quad \dot{\varphi}(t) = V/\varphi_0 \qquad \varphi_0 = \hbar/2e$$

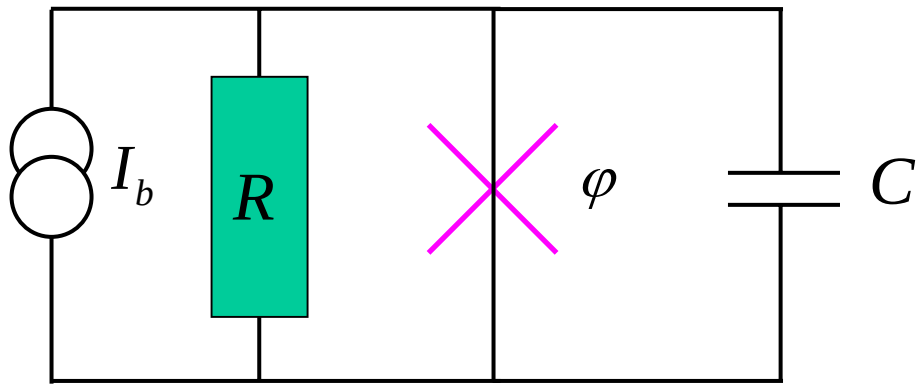
$$I_0 \rightarrow E_J/\varphi_0$$

Electrical circuit



RCSJ-Model





$$I_b = I_R + I_S + I_C$$

Josephson relations:

$$I_S = I_0 \sin(\varphi)$$

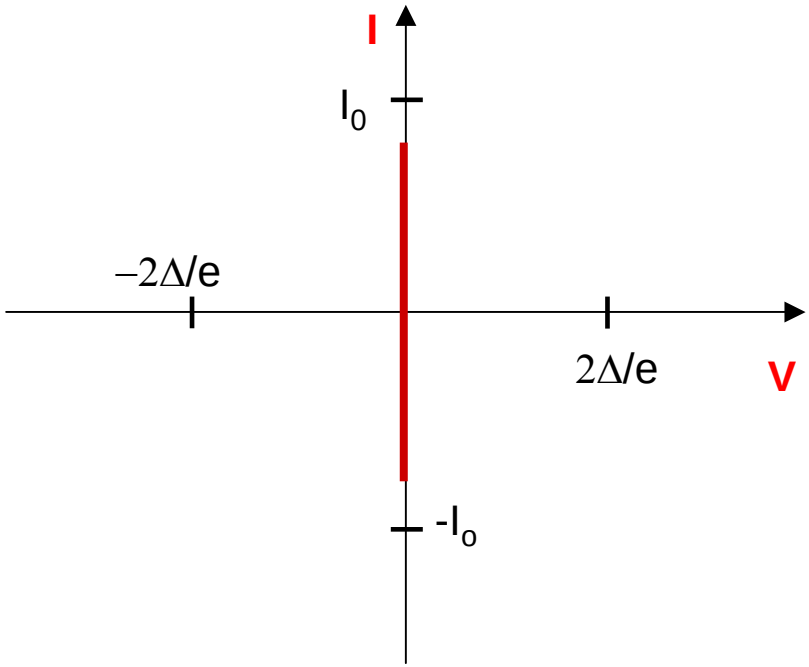
$$V = \varphi_0 \frac{d\varphi}{dt}$$

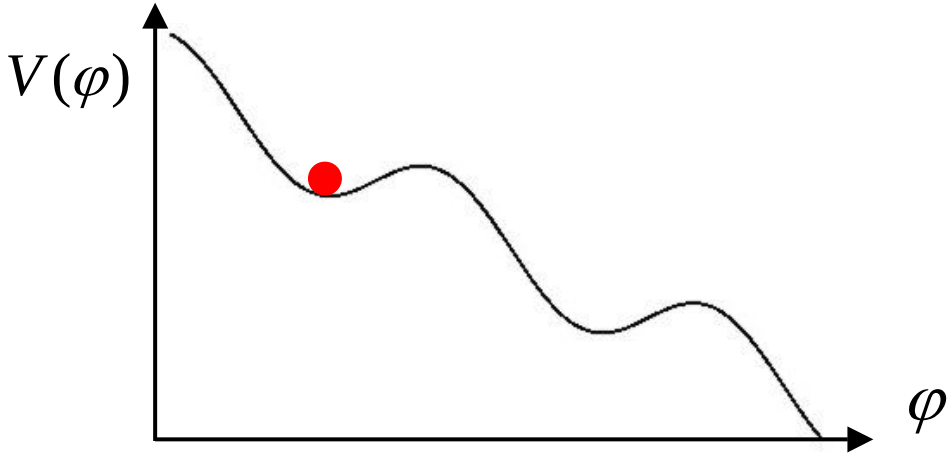
$$I_b = \frac{V}{R} + I_0 \sin(\varphi) + C\dot{V}$$

$$I_b = \frac{\varphi_0}{R}\dot{\varphi} + I_0 \sin(\varphi) + \varphi_0 \ddot{\varphi}$$

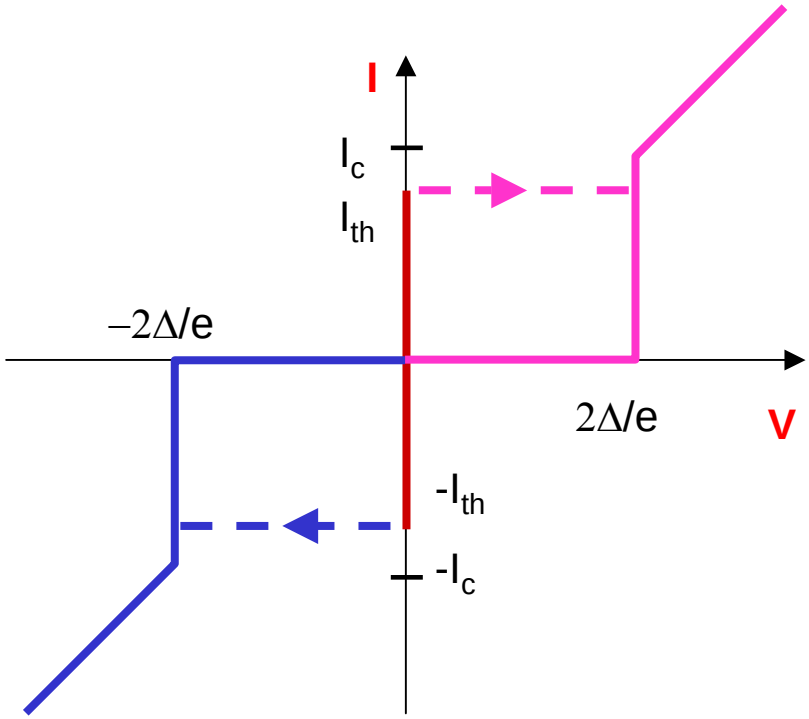
Motion of a classical particle in tilted periodic potential

Switching to finite voltage: Influence of thermal environment

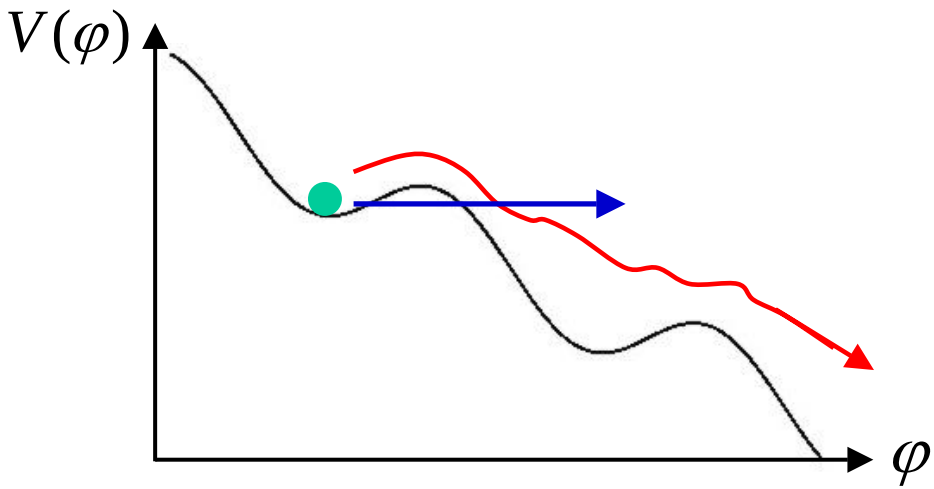




Switching to finite voltage: Influence of thermal environment



A voltage appears
if $|I|>I_{th}$



Escape by thermal activation

Or

Macroscopic tunneling

Classical noise: thermal activation

Translation rules:

$$\varphi_0^2 C \rightarrow M$$

$$\frac{1}{RC} \rightarrow \gamma$$

$$I_b = \langle I_b \rangle + \delta I$$

$$\langle I_b \rangle \varphi_0 \rightarrow F$$

$$\varphi_0 \delta I \rightarrow \xi$$

Classical phase dynamics:

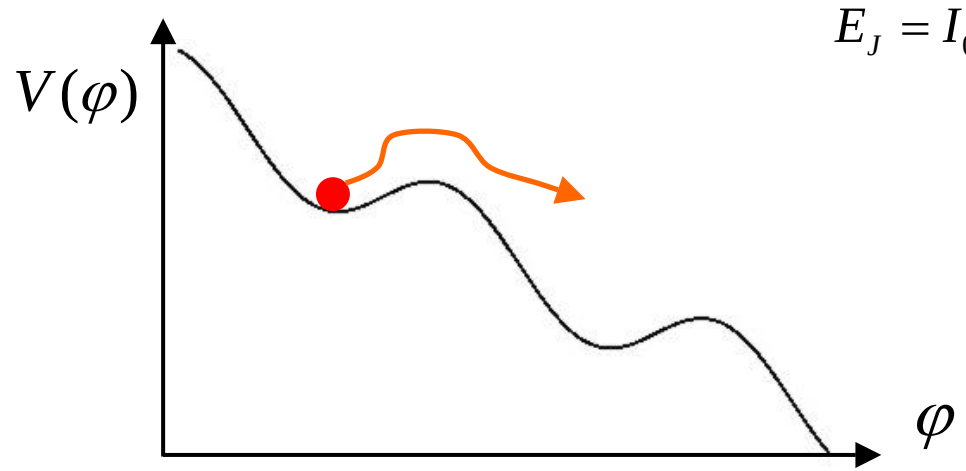
$$M\ddot{\varphi}(t) + \gamma\dot{\varphi}(t) + V'(\varphi) = \xi(t)$$

$$\langle \xi(t) \rangle = 0$$

$$\langle \xi(t)\xi(0) \rangle = 2M\gamma k_B T \delta(t)$$

$$V(\varphi) = -E_J \cos(\varphi) - F\varphi$$

$$E_J = I_0 \varphi_0$$

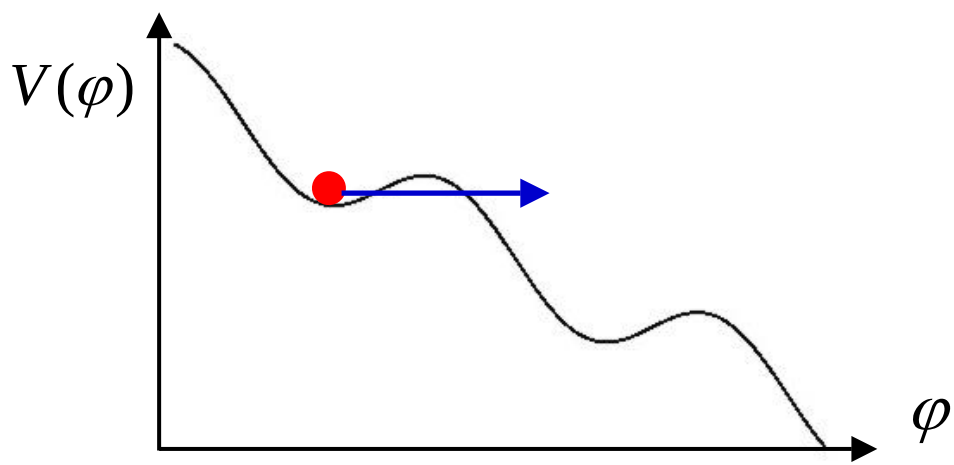


Quantum mechanics

$$[\hat{Q}, \hat{\varphi}] = ie$$

Macroscopic Quantum Tunneling

$$V(\varphi) = -E_J \cos(\varphi) - F\varphi$$



Experimental protocol

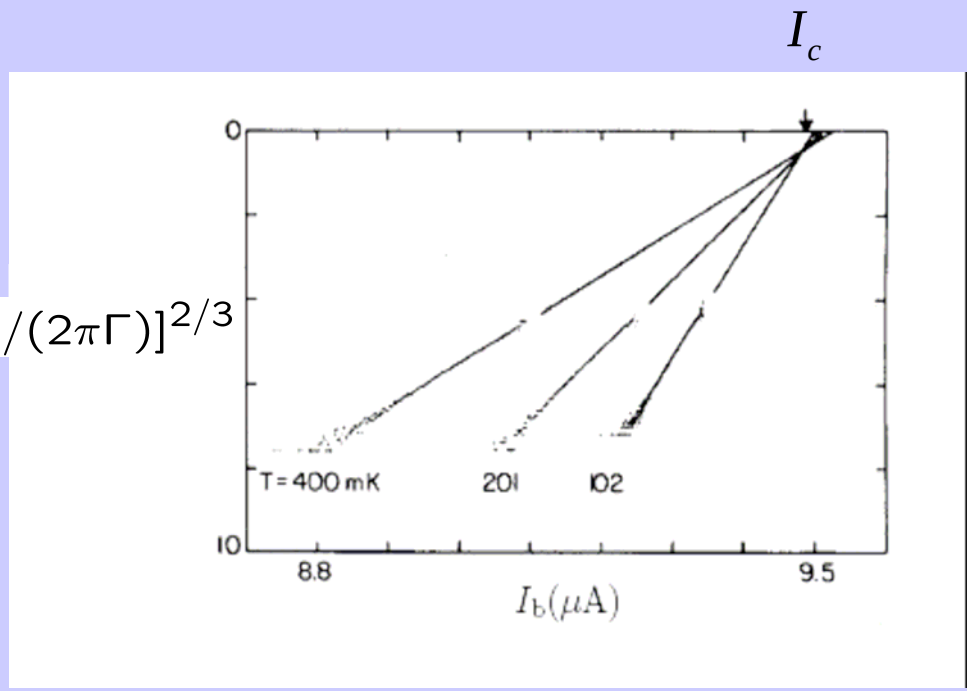
Cool down: 20mK....100mK

Determine $E_J, E_C : \; I_c \, , \; \omega = (\sqrt{2E_C E_J}/\hbar)(1-s)^{1/4} \, , \; s = I_b/I_0$

High temperature:

$$\Gamma \propto e^{-\beta V_b}$$

$$V_b \approx \frac{4\sqrt{2}}{3} E_J (1-s)^{3/2}$$



Experimental protocol:

Cool down: 20mK....100mK

Determine $E_J, E_C : \; I_c \, , \; \omega = (\sqrt{2E_C E_J}/\hbar)(1-s)^{1/4} \, , \; s = I_b/I_0$

High temperature, external microwaves (power P): resonant activation

Enhancement

$$\ln \Gamma(P)/\Gamma(0)$$

From the resonant enhancement:

Plasma frequency

Friction Q (width)

Experimental protocol:

Cool down: 20mK....100mK

Determine $I_c \ , \ \omega \ , \ Q$

Rise the bias current

Detect a switching via a voltage pulse

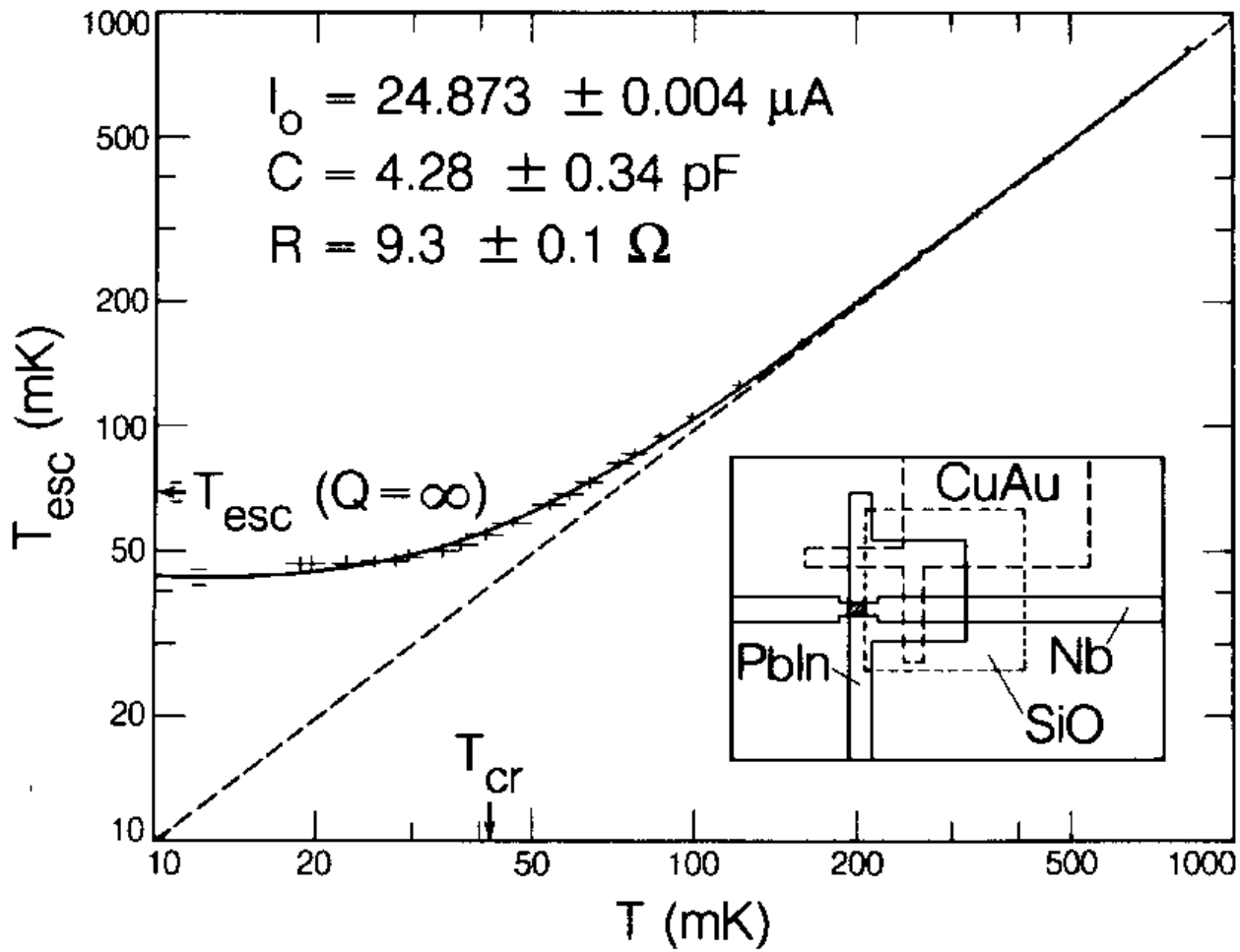
Repeat for many times

Make a histogram: #switchings vs bias current

Extract Tunneling rate

Experiment

$$\frac{const}{|\mathrm{Ln}(\Gamma)|}$$

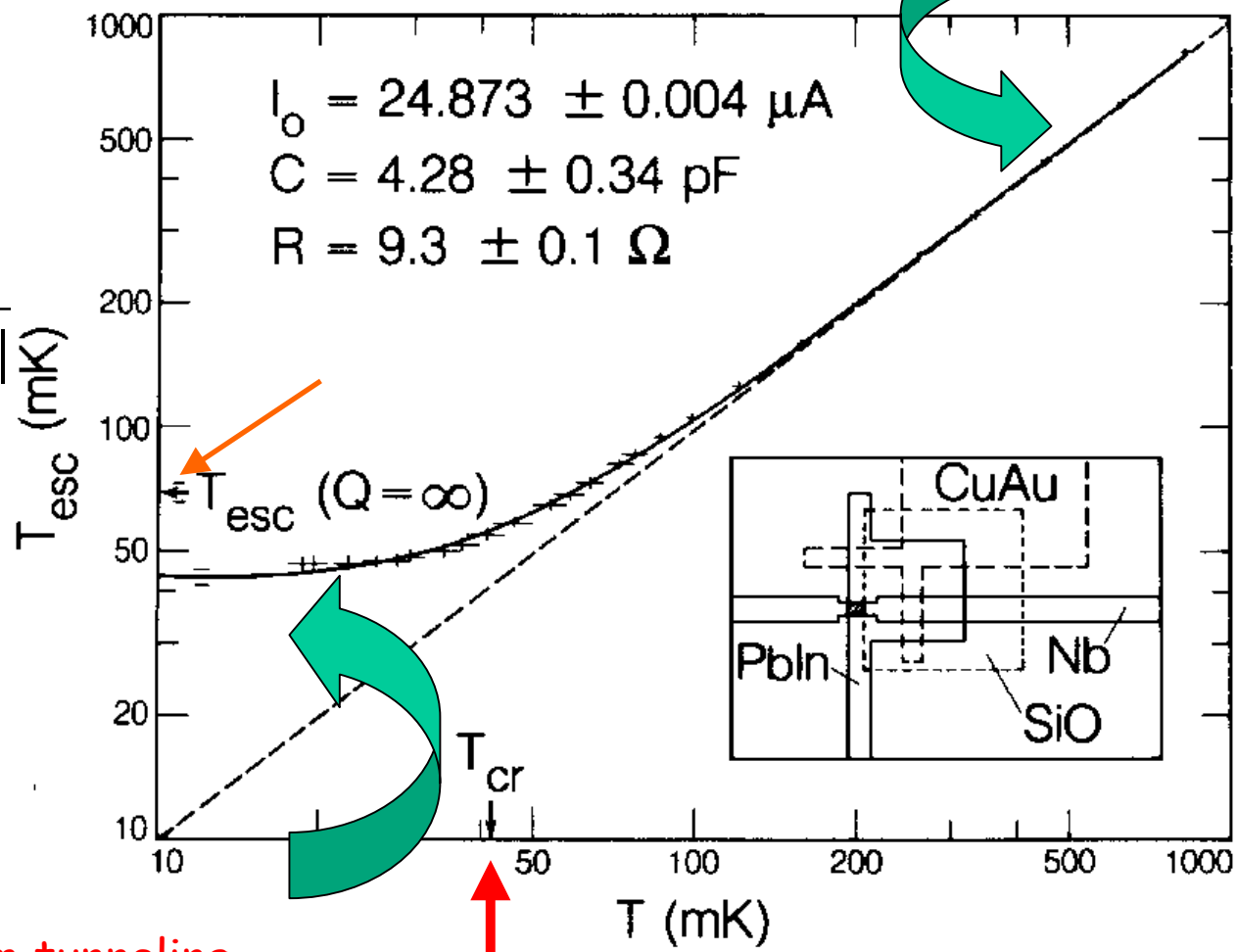


Devoret et al, 1988

Experiment

Thermal activation

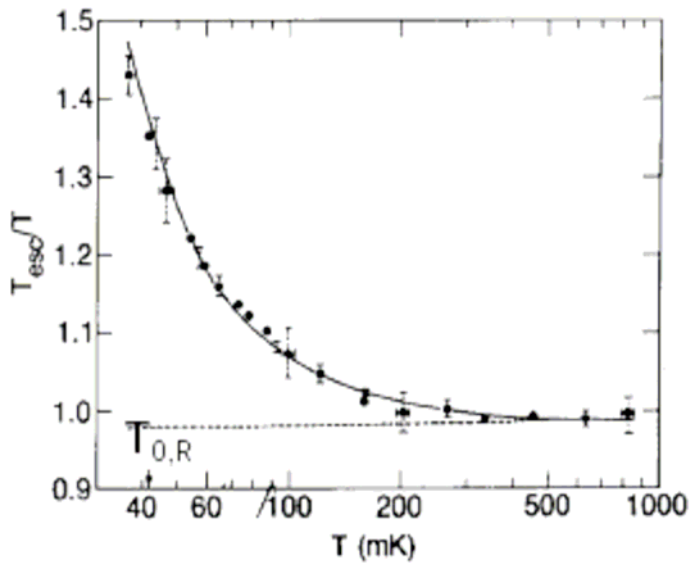
$$\frac{const}{|\mathrm{Ln}(\Gamma)|}$$



Quantum tunneling

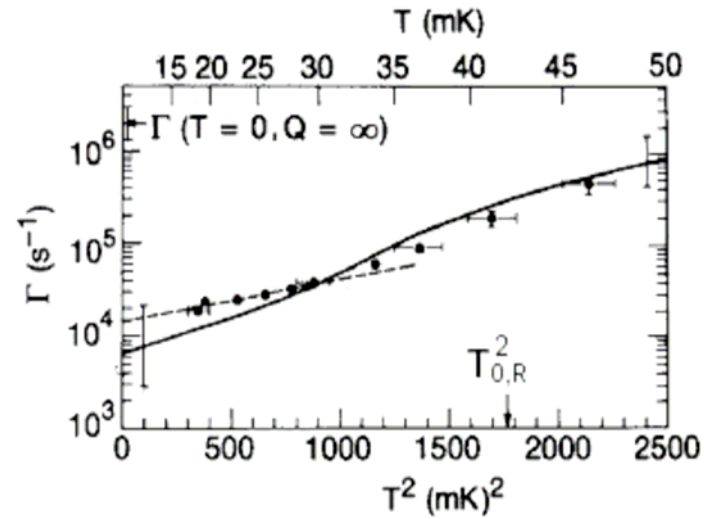
Experiment

Enhancement due to quantum fluctuations above crossover



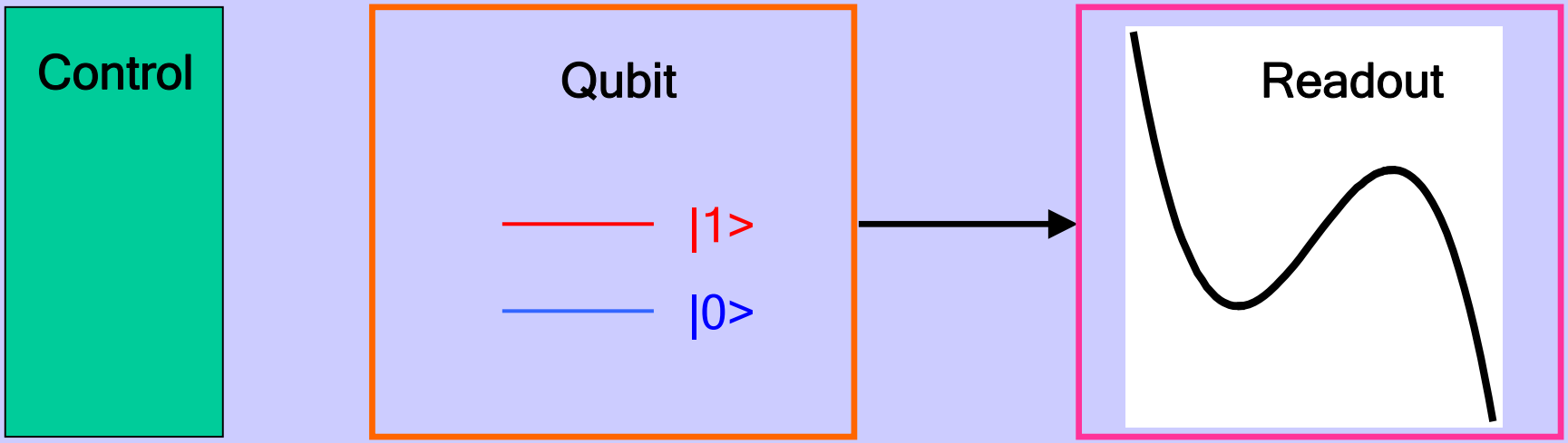
$$\frac{\Gamma}{\Gamma_{cl}}$$

Enhancement due to quantum fluctuations for very low T



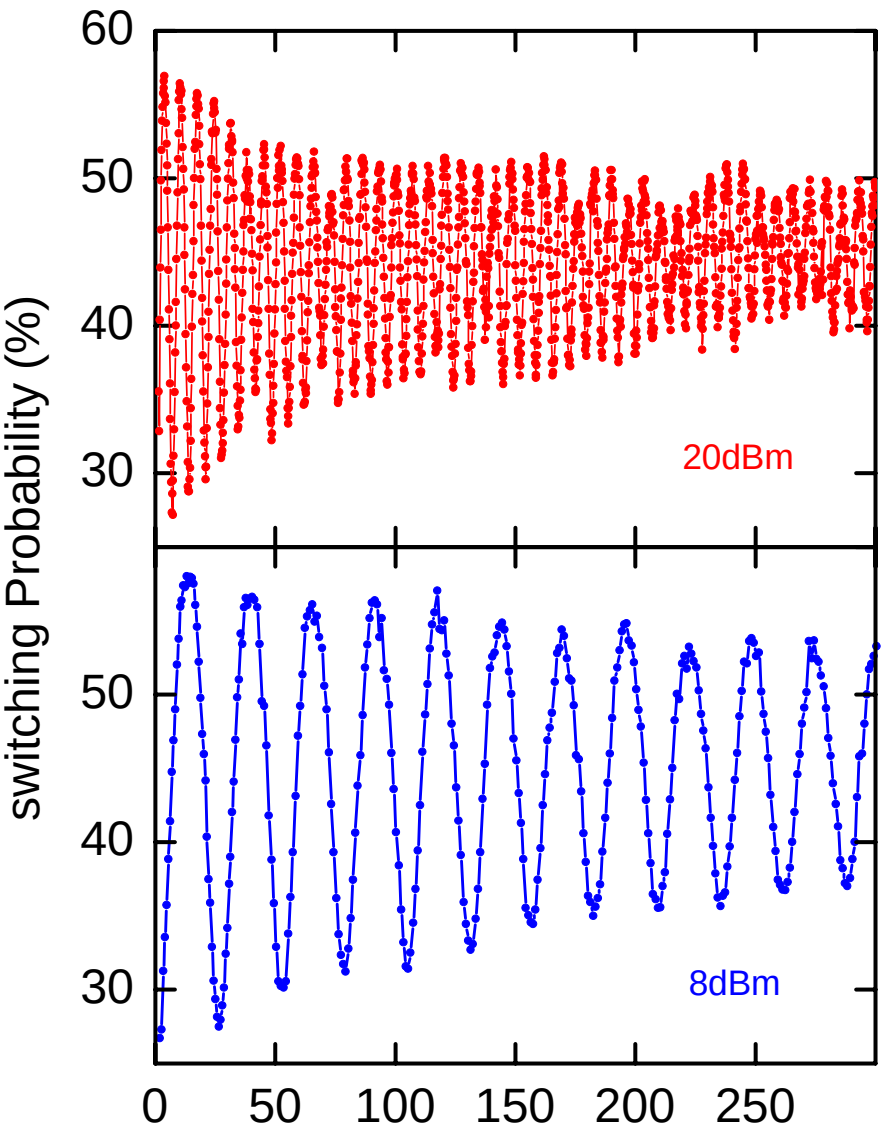
$$\frac{\Gamma(T)}{\Gamma(T=0)} \propto T^2$$

Designed Qubit Systems

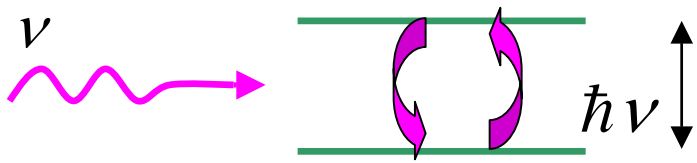


Exponential sensitivity to qubit state parameters

Superconducting Qubits

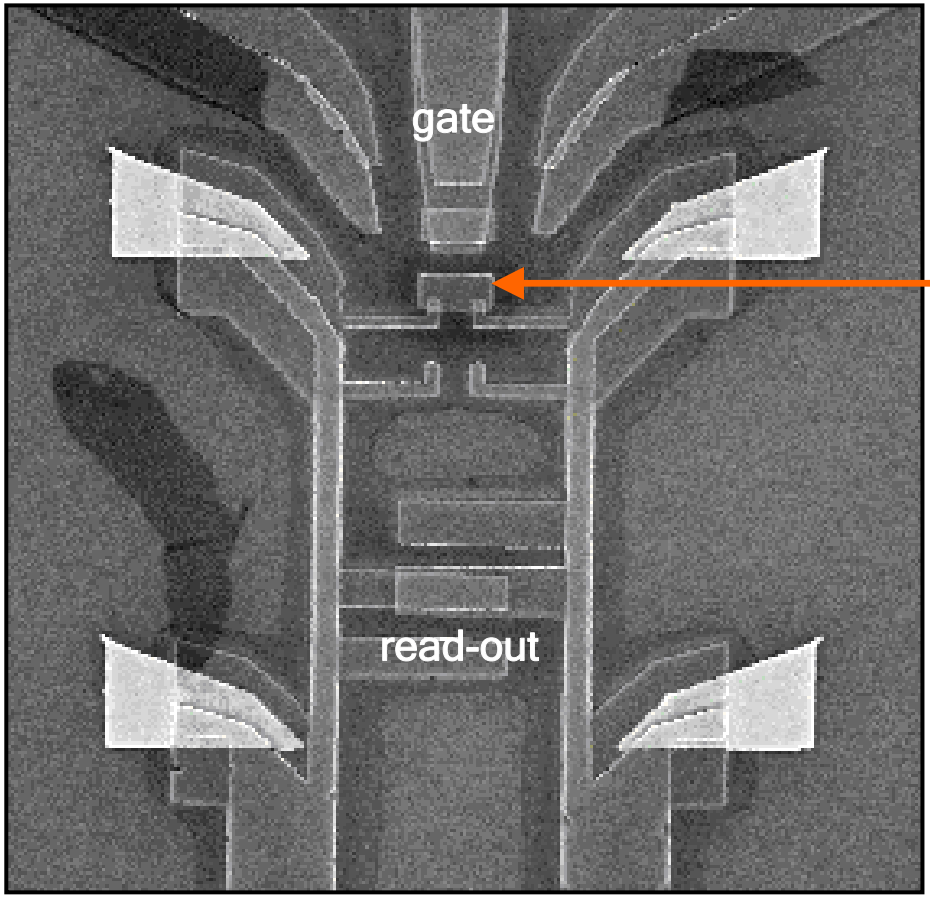


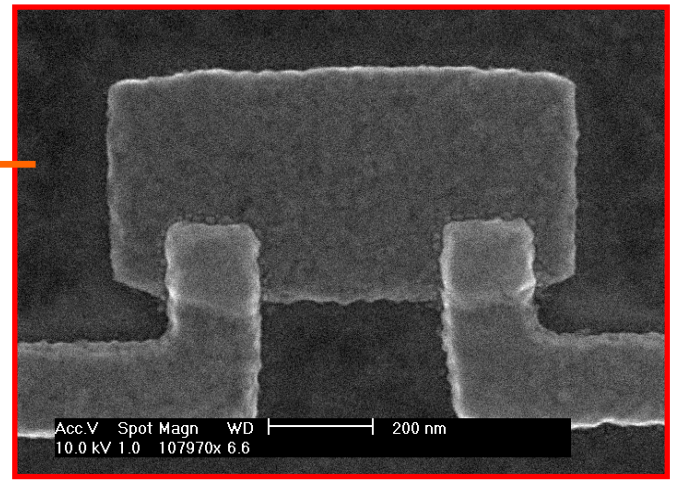
Rabi-oscillations in a two level atom



Vion, Esteve et al, Science 296, 886 (2002)

Artifial (mesoscopic) atom

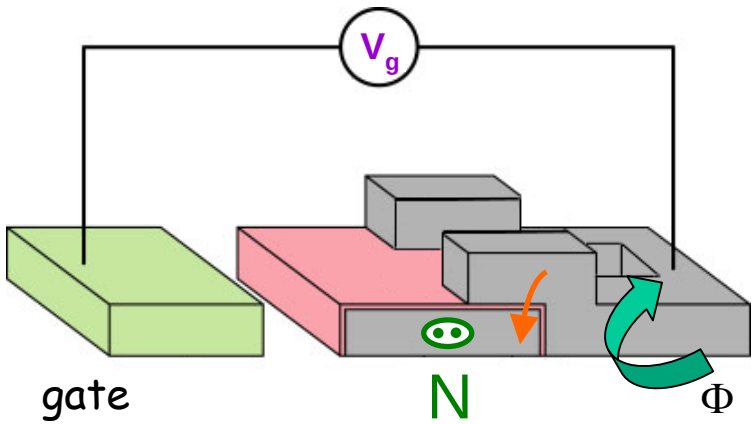


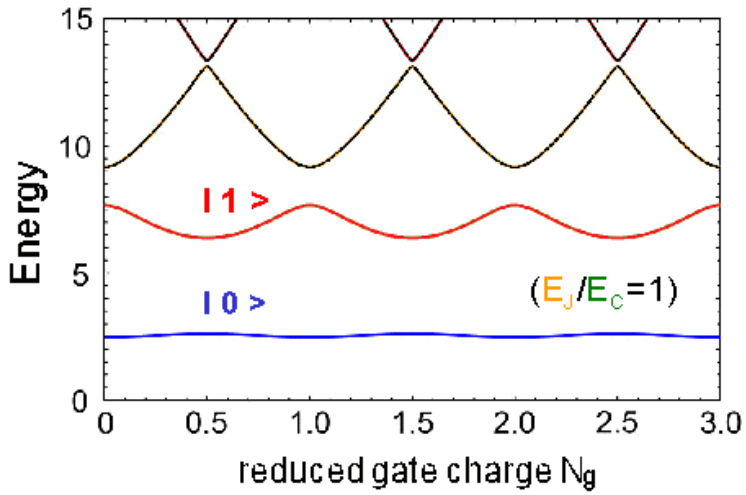


1 μm

Groupe Quantronique, CEA Saclay

Tunable atom

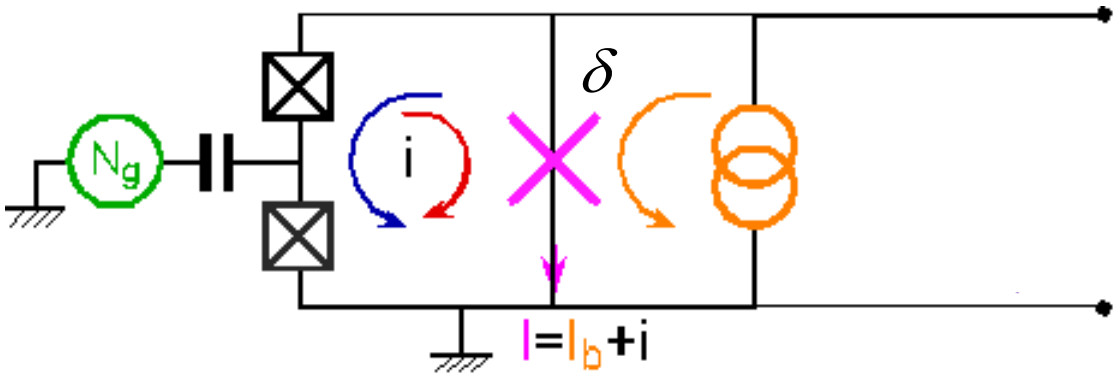


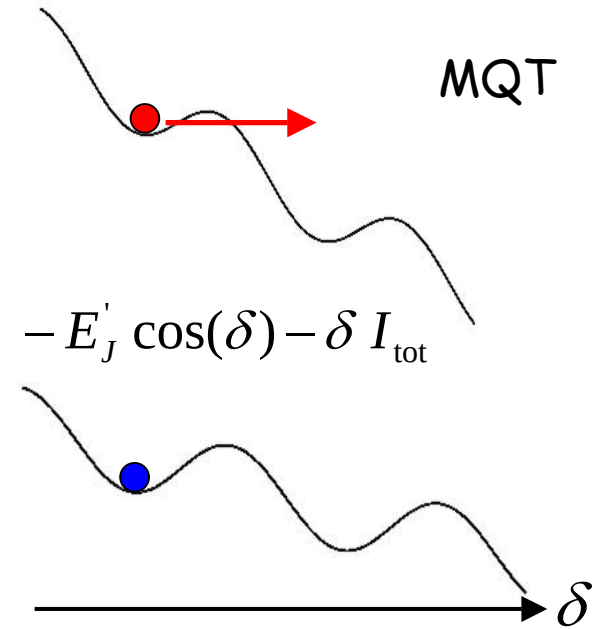


Superconducting circuit: decoupling from excitations

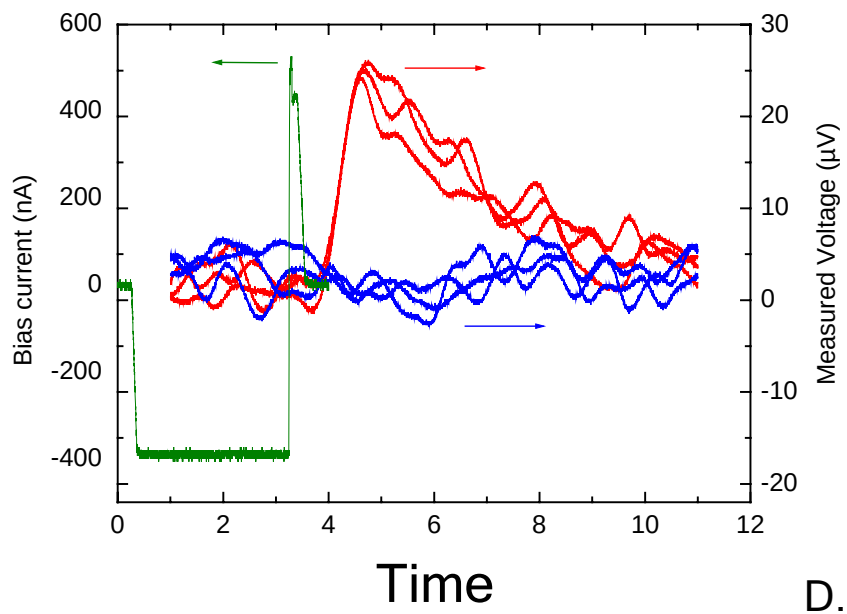
States |1> and |0> related to supercurrents i_1 and i_0

Readout: Quantronium





Bias current



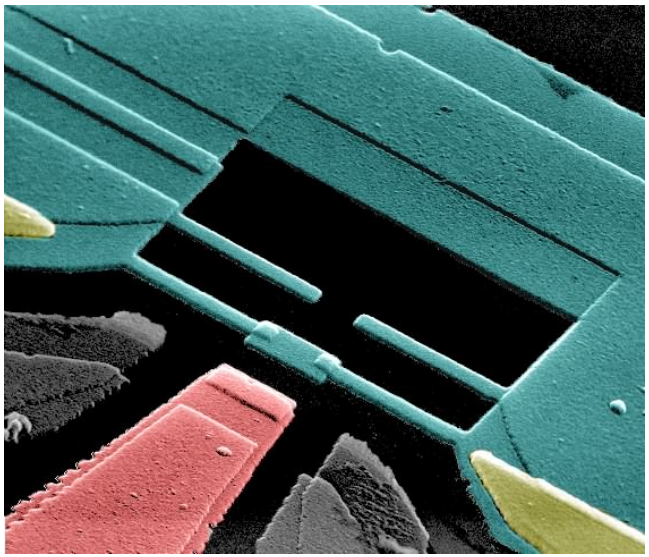
Voltage

D. Vion et al, Science (2002)

Quantronium (Charging regime)

Hamiltonian for N={0,1} excess Cooper pairs:

$$H = \underbrace{E_C\left(N_g - \frac{1}{2}\right)\sigma_z - E_J\cos\left(\frac{\delta+\phi}{2}\right)\sigma_x}_{\text{Qubit}} + \underbrace{\frac{p^2}{2m} - E_J^{'}\cos(\delta) - \frac{I}{I_0}\delta}_{\text{Read-out JJ}}$$

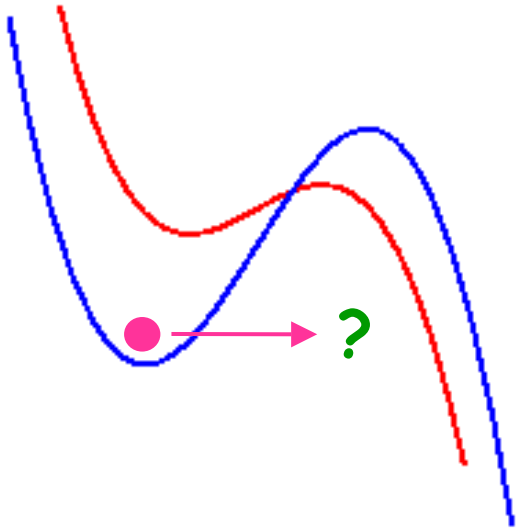


$N_g = C_g U/2e$ Gate voltage

$\phi = \Phi/(\hbar/2e)$ Magnetic flux

$N_g = \frac{1}{2}$, $\delta + \phi = \pi$ Crossing

Tunneling of a qubit: Crossing of surfaces



$$H = \begin{pmatrix} \frac{p^2}{2m} + V_+ & \Delta(\delta) \\ \Delta(\delta) & \frac{p^2}{2m} + V_- \end{pmatrix}$$

Flip: Smaller barrier → larger rate ?

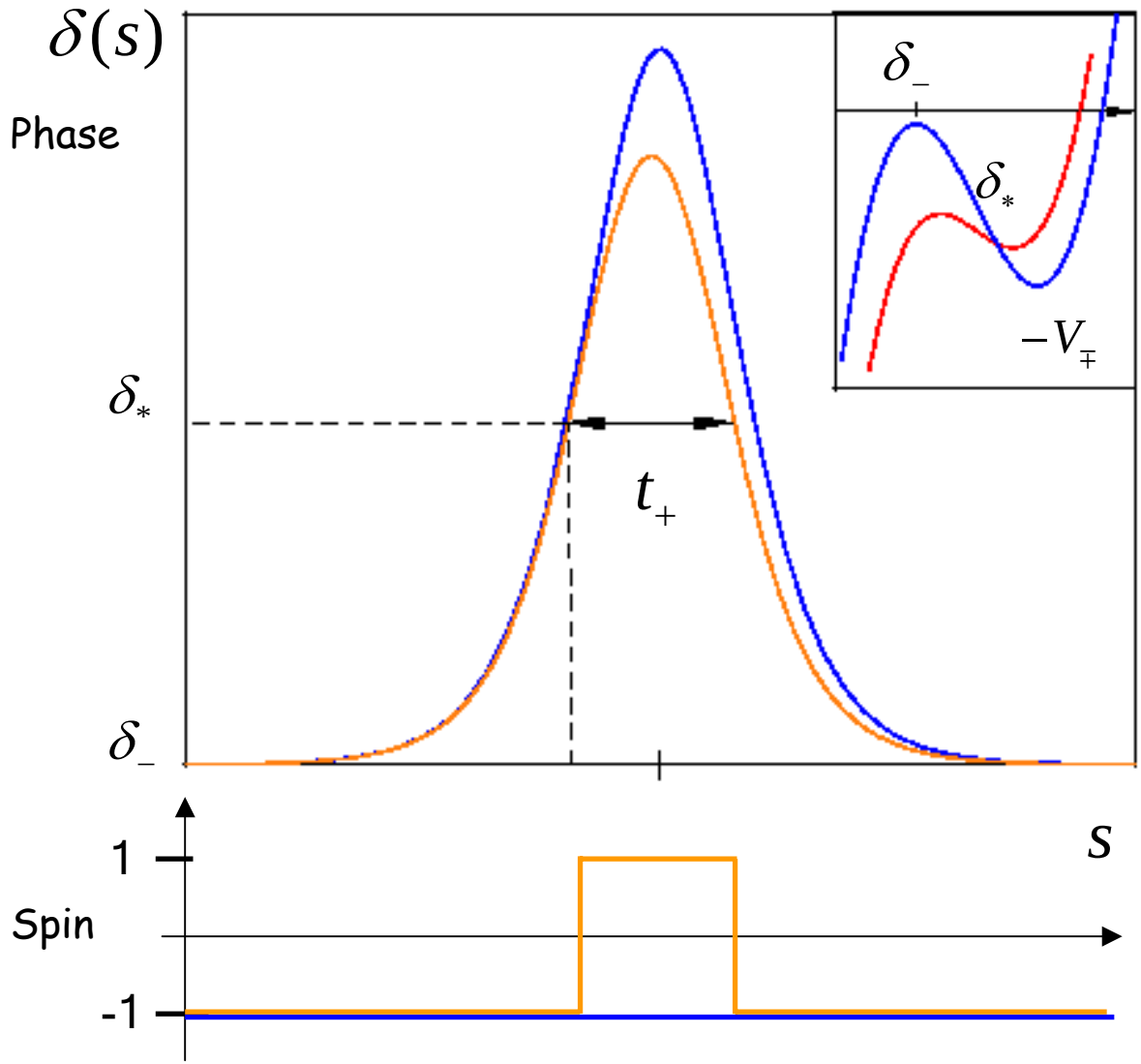
Landau-Zener transitions „under" the barrier: MQT of a Spin

JA et al, PRL 91, 016803 (2003)
Vion et al & JA, PRL 94, 057004 (2005)

Spin-flip tunneling

Simple bounce

Flip bounce

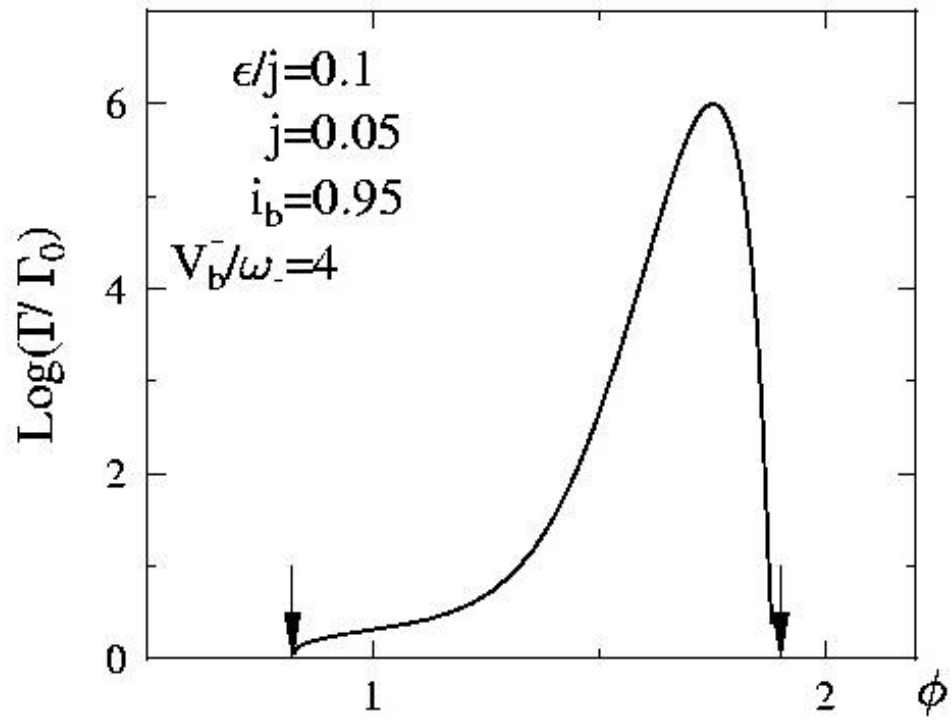


MQT-Flip Rate

Prediction: Rate enhancement:

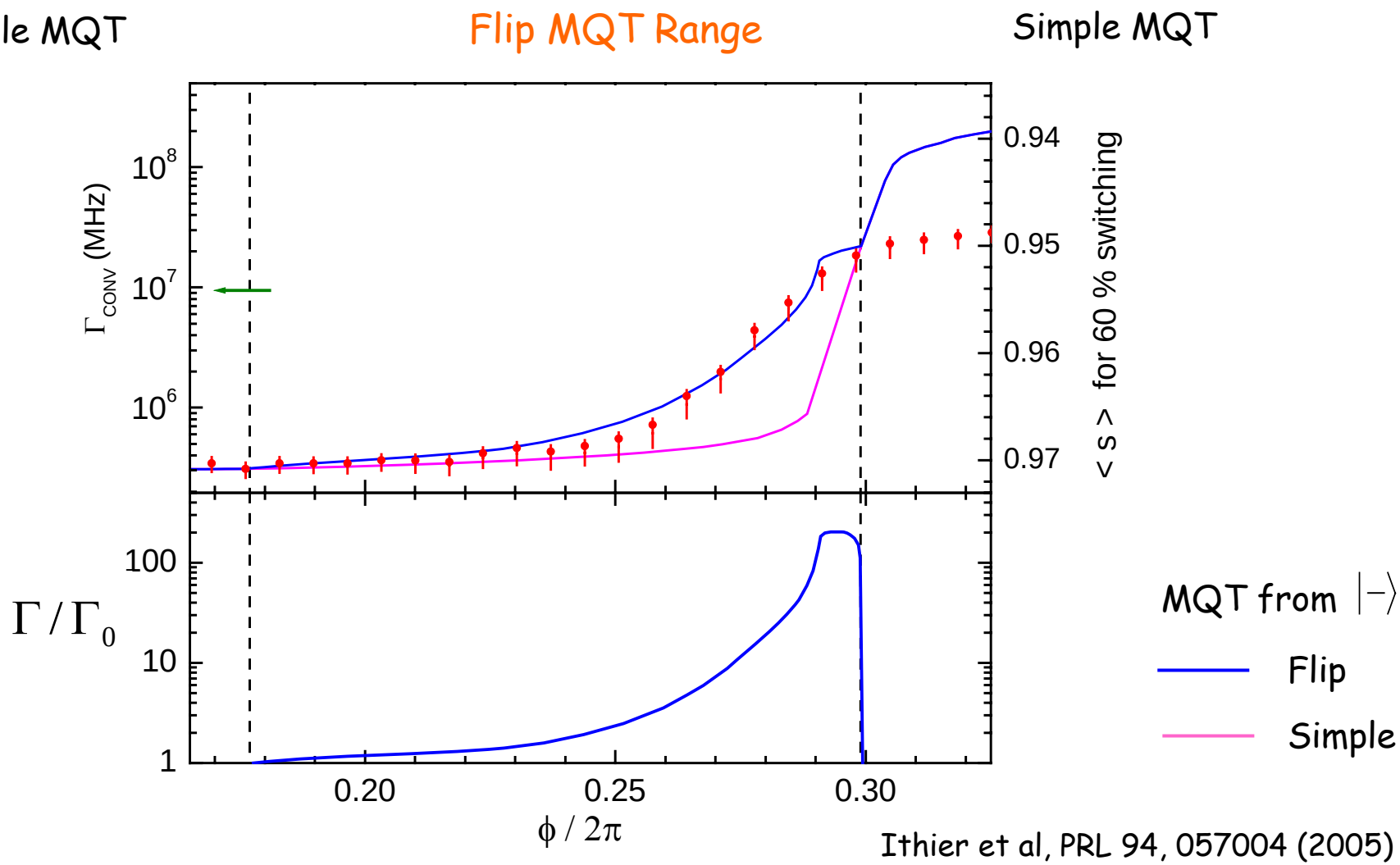
$$\Gamma = \Gamma_0 + \Gamma_F$$

Non-flip Flip



JA et al , PRL 016803 (2003)

Experiment: Rate Enhancement



Ithier et al, PRL 94, 057004 (2005)

Theory vs. Experiment

Summary Lecture 3:

- Josephson junction: fictitious particle
- switching rate measurements, crossover
- Spin-flip tunneling in Josephson qubits

Thanks