Macroscopic Quantum Tunneling and Coherence

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Set-up

Lecture 1: Basics and Theoretical Tools
   Goal: How to describe quantum tunneling

Lecture 2: Tunneling in Macroscopic Systems
   Goal: Understanding MQT and MQC

Lecture 3: Tunneling in Josephson Systems
   Goal: Understanding MQT and MQC in Josephson devices
Set-up

Lecture 1: Basics and Theoretical Tools

- Introduction
- Simple examples
- Transmission and decay rates
- ImF-technique
- Quantum Coherence

**Goal: How to describe quantum tunneling**

Lecture 2: Tunneling in Macroscopic Systems?

Lecture 3: Tunneling in Josephson Systems
Quantum tunneling is cool!

- 5 Nobel prizes in physics related to tunneling (semiconductors, superconductors, STM)
- Quantum tunneling: Google about 2 million hits
- Macroscopic quantum tunneling: Google about 500,000 hits
Quantum tunneling is cool!

- 5 nobel prizes in physics related to tunneling (semiconductors, superconductors, STM)
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1923, De Broglie:
Matter waves, in analogy to optics: penetration of forbidden regions
Tunneling in NH$_3$

Friedrich Hund 1927: „Zur Deutung der Molekülspektren“

F. Hund, Z. Phys. 40, 742 (1927) and 43, 805 (1927)
Alpha-Decay

\[ ^{240}_{94}\text{Pu} \rightarrow ^{236}_{92}\text{U} + \alpha = ^{4}_{2}\text{He} \]

\[ \frac{A}{Z} X \rightarrow \frac{A-4}{Z-2} Y^* + ^{4}_{2}\text{He} + E_\alpha \]

G. Gamow, Z. Phys. 51, 204 (1928)
J.R. Oppenheimer, Phys. Rev. 31, 80 (1928)

Quantum Tunneling
Barrier transmission: Scattering

\[ \phi_I(x) = e^{ikx} + r(E) e^{-ikx} \]
\[ \phi_{II}(x) = a e^{-\kappa x} + b e^{\kappa x} \]
\[ \phi_{III}(x) = t(E) e^{ikx} \]

\[ k = \sqrt{2mE/\hbar} \, , \quad \kappa = \sqrt{2m(V_b - E)/\hbar} \]
Barrier transmission: Scattering

\[ T(E) = |t(E)|^2 = \frac{4E(V_b - E)}{4E(V_b - E) + V_b^2 \sinh^2(\kappa L)} \]

\[ T(E) + R(E) = 1 \]
Barrier transmission: Deep tunneling

\[ V_b \gg E : \quad T(E) \propto e^{-2\sqrt{2m(V_b - E)L/\hbar}} \]

\[ W(E) = \int dx \sqrt{-2m(E - V_b)} = \int dx |p(x)| \]

Imaginary momentum: Euclidian action
Generalization: WKB

\[
\left[ \frac{\hat{p}^2}{2m} + V(x) - E \right] \psi(x) = 0 \quad \psi(x) = e^{i\tilde{W}(x)/\hbar}, \quad \tilde{W} = W_0 - i\hbar W_1 \ldots
\]

Smooth and high barrier!
(short wavelengths: geometric optics)

\[
W_0(x,a) = \int_a^x dy \ p(y)
\]

\[
T_{\text{WKB}}(E) = e^{-W(E)/\hbar}, \quad W(E) = 2|W_0(b,a)|
\]
Incoherent tunneling from a reservoir

Outgoing flux:  \( j_{\text{out}}(E) = T(E) v_{\text{out}}(E) \rho(E) P(E) \)

Total rate:  \( \Gamma = \frac{1}{Z} \int_{0}^{\infty} \frac{dE}{2\pi \hbar} T(E) P(E) \)

Has dimension of a frequency

\( v_{\text{out}}(E) \rho(E) = \frac{dE}{\hbar E} \times \frac{dk}{2\pi dE} = \frac{1}{2\pi \hbar} \)
Example: Scanning tunneling microscope

SiC (0001) 3×3 surface
Tunneling current

Temperature = 0:
P(E) = Fermi-function = Step-function

\[ I = e \int_{E_F - eV}^{E_F} dE \, T(E) \, v(E) \rho(E, x = 0) \]

Integrand approx. constant near Fermi energy

\[ I = \frac{V}{R_T(d)} \]

Tunneling resistance:

\[ \frac{1}{R_T(d)} = e \rho(E_F, x = 0) \, v_F \, e^{-2d \sqrt{2mV_0} / h} \]

Exponential sensitivity
More powerful: ImF

Problem: Energy averaging may be complicated (dissipative systems)

Idea:
Consider unstable system with quasi-bound states $\epsilon_n = E_n - i\hbar \Gamma_n / 2$

\[
Z = \sum_{n=0}^{\infty} e^{-\beta \epsilon_n}
\approx \sum_{n=0}^{\infty} e^{-\beta E_n} + i \frac{\hbar \beta}{2} \sum_{n=0}^{\infty} \Gamma_n e^{-\beta E_n}
= Z_0 \left(1 + i \frac{\hbar \beta}{2} \langle \Gamma_n \rangle \right)
\]

with $F = (-1/\beta) \ln Z$

$\Gamma \propto \text{Im} F$
More powerful: \( \text{ImF} \)

Comparison with WKB provides:

\[
\Gamma = -\phi_T \text{Im} F
\]

\[
\phi_T = \frac{2}{\hbar} \begin{cases} 
T_0/T & \text{for } T \geq T_0 \\
1 & \text{for } T \leq T_0
\end{cases}
\]

Crossover temperature \( T_0 = \frac{\hbar \omega_b}{2\pi k_B} \) (non-dissipative)
How to obtain the free energy ????
Path integrals

Feynman:

\[ \langle q_f | e^{-iHt/\hbar} | q_i \rangle = \int_{q(0)=q_i}^{q(t)=q_f} D[q] e^{iS[q]/\hbar} \]

“Sum over all paths”
Path integrals in imaginary time

\[ e^{-i\frac{Ht}{\hbar}} \xrightarrow{t \rightarrow -i\hbar \beta} e^{-\beta H} \]
Path integrals in imaginary time

Partition function: \[ Z = \text{Tr}\{e^{-\beta H}\} = \int \mathcal{D}[q] \ e^{-\frac{SE[q]}{\hbar}} \]

\[ SE[q] = \int_{0}^{\hbar \beta} d\tau \left[ \frac{m\dot{q}^2}{2} + V(q) \right] \]

Sum over all periodic paths in \( \tau \in [0, \hbar \beta] \)

running in the inverted potential
How to evaluate?

For sufficiently high barriers:
Integral dominated by least action paths
(but remember that we are interested in “ImF”)

Least action paths: \( m\ddot{q} - V'(q) = 0 \), \( q(0) = q(\hbar \beta) \)

Fluctuations: \[ Z \approx \sum_{\alpha} D_{\alpha} \ e^{-S[q_{\alpha}]/\hbar} \]

Up to second order: Gaussian integrals
Metastable well: high temperature

\[ \hbar \beta < \frac{2\pi}{\omega_b} \]

Stable: \( S[q = q_0] = 0 \)

Unstable: \( S[q = q_b] = \beta V_b \)
Metastable well: high temperature

Thermal activation + quantum fluctuations

\( \hbar \beta < 2\pi / \omega_b \)

Stable: \( S[q = q_0] = 0 \)

Unstable: \( S[q = q_b] = \beta V_b \)

Gaussian fluctuations
In analytic continuation

\[
\Gamma = \frac{\omega_0 \sinh(\omega_0 \hbar / 2)}{2\pi \sin(\omega_b \hbar / 2)} e^{-\beta V_b}
\]

Imaginary part
Metastable well: Low temperature

Quantum tunneling

$\hbar \beta < 2\pi / \omega_b$

$\hbar \beta > 2\pi / \omega_b$

$\Gamma = \frac{\omega_0}{2\pi} f_q e^{-\beta V_b}$

$cubic: S_B = \frac{36 V_b}{5 \omega_0}$
Metastable well: Low temperature

\( \hbar \beta < 2\pi / \omega_b \)

\( \hbar \beta > 2\pi / \omega_b \)

Quantum tunneling

Crossover \( \omega_b \hbar \beta = 2\pi \)

cubic: \( S_B = \frac{36 \, V_b}{5 \, \omega_0} \)
\[ \frac{\text{const}}{|\ln(\Gamma)|} \]

Crossover
Crossover

Thermal activation

Quantum tunneling

\[ \frac{\text{const}}{|\ln(\Gamma)|} \]

\[ T_{\text{esc}} \text{ (mK)} \]

\[ T \text{ (mK)} \]

\[ T_0 \]
Coherent tunneling
Double well
Coherent tunneling

Ground state

Localized basis: $|0_L\rangle, |0_R\rangle$

2-state Hamiltonian: $H = \begin{pmatrix} \hbar \omega / 2 & \hbar \Delta \\ \hbar \Delta & \hbar \omega / 2 \end{pmatrix}$
Coherent tunneling

Ground state doublet

Localized basis:  |0L⟩  |0R⟩  

2-state Hamiltonian:  \[ H = \begin{pmatrix} \frac{\hbar \omega}{2} & \hbar \Delta \\ \hbar \Delta & \frac{\hbar \omega}{2} \end{pmatrix} \]

Eigenbasis:  

Energies:  \[ E_{\pm} = \frac{\hbar \omega}{2} \pm \hbar \Delta \]
Coherent tunneling dynamics

Initial state:

\[ |\psi(0)\rangle = (|0_s\rangle - |0_a\rangle)/\sqrt{2} \equiv |0_R\rangle \]

Time evolution:

\[ |\psi(t)\rangle = e^{-i\omega t/2} (e^{i\Delta t}|0_s\rangle - e^{-i\Delta t}|0_a\rangle)/\sqrt{2} \]

Overlap:

\[ \langle\psi(0)|\psi(t)\rangle = \cos(\Delta t) \]

Spectroscopy measures \( \Delta \)
Bloch sphere

Initial state:

$$|\psi(0)\rangle = (|0s\rangle - |0a\rangle)/\sqrt{2} \equiv |0R\rangle$$

In eigenstate basis:

$$H = \frac{\hbar \omega}{2} I + \hbar \Delta \sigma_z$$

Particle in a magnetic field in z-direction
Tunnel splitting from partition function

\[ Z = \text{Tr}e^{-\beta H} = \int \mathcal{D}[q] \ e^{-S_E[q]/\hbar} \]

Least action paths in inverted double well for very low T:

![Graph showing least action paths in an inverted double well](image)

**Instanton**
Tunnel splitting from partition function

\[ Z = \text{Tr} e^{-\beta H} = \int \mathcal{D}[q] \ e^{-S_E[q]}/\hbar \]

\(\hbar/\beta\) very large: Family of paths with \(n\) instanton/anti-instanton pairs, \(n=0,1,2,...\)
Tunnel splitting from partition function

\[ Z = \text{Tr} e^{-\beta H} = \int \mathcal{D}[q] \ e^{-S_E[q]}/\hbar \]

Sum over all pairs and their fluctuations in a time ordered manner: (zero mode!)

\[ Z = e^{-\beta \hbar \omega/2} \left( e^{\beta \hbar \Delta} + e^{-\beta \hbar \Delta} \right) \]

\[ \Delta \propto \omega_0 e^{-W(E=0)/2\hbar} \]

Instanton action
External driving: Rabi oscillations

$$A \sin(\Omega t) \sigma_x$$

Microwave field in resonance with energy splitting:

Oscillations with frequency \( \propto A \)
Summary Lecture 1:

- Coherent and incoherent tunneling
- Transmissions and averaged transmissions (decay rates)
- Partition function: Euclidian path integral (inverted pot.)
  - incoherent decay: ImF, bounce orbit (unstable)
  - coherent tunneling: instanton

Thanks
Set-up

Lecture 1: Basics and tools

- Introduction
- Simple examples
- WKB-approximation for transmission probabilities
- Decay rates and ImF-technique
- Quantum Coherence

Lecture 2: Tunneling in Macroscopic Systems?

- MQC and MQT?
- Dissipative tunneling
- Nanomagnets: coherent and incoherent tunneling
- SQUIDS
- BEC

**Goal:** Understanding MQT and MQC
So far: Tunneling of microscopic objects

Is tunneling observable for systems with macroscopically many degrees of freedom?

Is quantum mechanics valid for these systems?

And if we assume yes: How should we describe them?

Why is tunneling not observable in our world?

MQT, MQC: Tunneling of collective degrees of freedom
These systems are embedded in a surrounding they interact with the macroscopic world

Open systems: energy exchange feel fluctuating forces
Classical: Open systems

Energy flow: Relaxation

Stochastic scattering: Fluctuations

Coupling constant

Temperature
Open quantum systems

System + reservoir: reduced density

\[ \rho(t) = \text{Tr}_R \left\{ e^{-iHt/\hbar} W(0) e^{iHt/\hbar} \right\} \]

\[ \rho_\beta = \frac{1}{Z} \text{Tr}_R \left\{ e^{-\beta H} \right\} \]
Caldeira-Leggett-Model: (Magalinskii [1959], Ullersma [1966], Zwanzig [1973])

\[ H = H_S + H_I + H_B \]

\[ H_I + H_B = \sum_{i=1}^{\infty} \left[ \frac{p_i^2}{2m_i} + \frac{m_i \omega_i^2}{2} \left( x_i - \frac{c_i}{m_i \omega_i^2} q \right)^2 \right] \]

Harmonic oscillator bath with continuum of modes, thermal distribution

**Classical**: generalized Langevin equation for system

\[ M\ddot{q} + M \int_0^\infty ds \ \gamma(t-s)\dot{q}(s) + V'(q) = \xi(t) \]

Friction kernel  
Noise
Caldeira-Leggett-Model: (Magalinskii [1959], Ullersma [1966], Zwanzig [1973])

Friction determined by spectral bath density:

\[ \gamma(t) = \frac{2}{M} \int_0^\infty \frac{d\omega I(\omega)}{\pi \omega} \cos(\omega t) \]

Noise determined by dissipation-fluctuation theorem:

\[ \langle \xi(t)\xi(s) \rangle = M k_B T \gamma(t - s) \]

Markovian friction (ohmic, Johnson-Nyquist):

\[ \langle \xi(t)\xi(s) \rangle = 2M k_B T \gamma(t - s) \]

Influence of the bath given by

\[ T, I(\omega) \]

Gaussian statistics of the bath fluctuations
Quantization: Path integrals

Path integral in imaginary time after carrying out the Gaussian integrals:

\[ Z = \int \mathcal{D}[q] \ e^{-S_E[q]/\hbar} \ e^{-\Phi[q]/\hbar} \]
Macroscopic quantum tunneling

Crossover:

\[ T_0 = \frac{\hbar \omega_\gamma}{k_B 2\pi} \quad \omega_\gamma^2 + \omega_\gamma \gamma(\omega_\gamma) - \omega_b^2 = 0, \quad \omega_\gamma > 0 \]

Fourier transform of \( \gamma(t) \)

Friction suppresses quantum effects

Ohmic friction: \( \bar{\gamma}(\omega) = \gamma \)
Macroscopic quantum tunneling rates

Above crossover

\[ \Gamma = \frac{\omega_0 \omega_\gamma}{2\pi \omega_b} \left[ \prod_{n=1}^{\infty} \frac{\nu_n^2 + \omega_0^2 + \nu_n \tilde{\gamma}(\nu_n)}{\nu_n^2 - \omega_b^2 + \nu_n \tilde{\gamma}(\nu_n)} \right] e^{-\beta V_b} \]

Below crossover

\[ \Gamma = \sqrt{\frac{S_B}{2\pi \hbar}} \left[ \prod_{n=1}^{\infty} \frac{\lambda_n^{(0)}}{\lambda_n^{(b)}} \right] e^{-S_B/\hbar} \]

\[ \gamma \text{ small : } S_B \approx \frac{36 V_b}{5 \omega_0} [1 + O(\gamma)] \]

\[ \gamma \text{ large : } S_B \approx 3\pi \gamma V_b / \omega_0^2 \]
Example: Tunneling of magnetization
Tunneling of magnetization in Fe₈ clusters

\[ H = -D S_z^2 + E(S_x^2 - S_y^2) - gS_z B_z \]

Anisotropy: induces tunneling
MQT in iron clusters

Sangregorio et al, PRL 78, 4645 (1997)
Influence of the environment: phonon induced transitions

\( \text{Mn}_{12} \)

Bokacheva et al, PRL 85, 4803 (2000)
Reduced real-time dynamics: Master equations
Master/Redfield-Equation

2. order perturbation theory in $H_f$

$$i\hbar \frac{d\rho(t)}{dt} = [H_S, \rho(t)] + \gamma^2 \mathcal{R}[\rho(t)]$$

- Various types: CL-Master equation, Lindblad, ...
- numerically efficient
- weak friction: $\gamma \hbar / \beta \ll 1$
- fast bath modes: $\omega_0 \ll \omega_c$
Dephasing and relaxation

\[ H = \begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix} \]
Dephasing and relaxation

\[ H = \begin{pmatrix} \epsilon + \xi(t) & 0 \\ 0 & -\epsilon - \xi(t) \end{pmatrix} \]

**Dephasing:** Noise couples to the eigenstates
Affects only the non-diagonal elements of the density
Dephasing and relaxation

\[ H = \begin{pmatrix} \epsilon + \xi(t) & \Delta \\ \Delta & -\epsilon - \xi(t) \end{pmatrix} \]

**Dephasing**: Noise couples to the eigenstates
Affects only the non-diagonal elements of the density

**Relaxation**: transitions, leads to equilibrium
Affects also the diagonal elements

\[ e^{-\beta E_n} \]
Thermal bath

Statistics of the bath coupling operator $X = \sum_{i=1}^{\infty} c_i x_i$

Spectral power:
$$S(\omega) = \frac{1}{2} \langle \{X(t), X(s)\} \rangle_\omega = \hbar I(\omega) \coth(\beta \hbar \omega / 2)$$

$$\langle \sigma_\pm(t) \rangle \to \langle \sigma_\pm(0) \rangle e^{\mp i 2 \epsilon t / \hbar} e^{-\gamma_{\text{dep}} t}$$

$$\langle \sigma_z(t) \rangle = \sigma_z^\infty + [\langle \sigma_z(0) \rangle - \sigma_z^\infty] e^{-\gamma_{\text{rel}} t}$$

$$\gamma_{\text{dep}} = \frac{\gamma_{\text{rel}}}{2} + \eta^2 S(\omega = 0)$$
$$\gamma_{\text{rel}} = \eta^2 S(\omega = 2\epsilon)$$

Dephasing rate
Relaxation rate
Thermal bath

Statistics of the bath coupling operator \( X = \sum_{i=1}^{\infty} c_i x_i \)

Spectral power:
\[
S(\omega) = \frac{1}{2} \langle \{X(t), X(s)\} \rangle_\omega = \hbar I(\omega) \coth(\beta \hbar \omega/2)
\]

\[
\langle \sigma_{\pm}(t) \rangle \rightarrow \langle \sigma_{\pm}(0) \rangle e^{\mp i 2 \epsilon t/\hbar} e^{-\gamma_{\text{dep}} t}
\]

\[
\langle \sigma_z(t) \rangle = \sigma_z^\infty + [\langle \sigma_z(0) \rangle - \sigma_z^\infty] e^{-\gamma_{\text{rel}} t}
\]

\[
\gamma_{\text{dep}} = \frac{\gamma_{\text{rel}}}{2} + \eta^2 S(\omega = 0)
\]

\[
\gamma_{\text{rel}} = \eta^2 S(\omega = 2\epsilon)
\]

\[\rightarrow 0 \text{ for } T \rightarrow 0\]
Examples

Examples

Examples
Example 1: **Superconducting Quantum Interference Device**

**Rf-SQUID**

\[ \varphi = -2\pi\left(\Phi/\varphi_0 - n\right), \quad \Phi = \Phi_x + LI_S \]

\[ \varphi_0 = \hbar/2e \]

Dynamical variable is now \( \Phi \)

**Potential:**

\[ U(\Phi) = \frac{(\Phi - \Phi_x)^2}{2L} - E_J \cos(\Phi/\varphi_0) \]

Lecture 3

\[ \beta_L = LE_J/\varphi_0^2 = 10, \quad \Phi_x = \pi\varphi_0 \]
DC-SQUID

Less sensitive to noise

Rabi oscillations for the ground state doublet

A. Lupascu, ENS

Example 2: Bose Einstein Condensate

Wineland et al
Coherent Tunneling of a BEC

\[ z = \frac{N_l - N_r}{N}, \quad \phi = \phi_r - \phi_l \]

Two-mode approximation:

\[ \dot{z} = -\sqrt{1 - z^2} \sin \phi \]
\[ \dot{\phi} = \Lambda z + \frac{z}{\sqrt{1 - z^2}} \cos \phi \]

Non-rigid pendulum with angular momentum \( z \)

\[ z(0) < z_c \quad z(0) > z_c \]

M. Albiez et al, PRL 95, 010402 (2005)
Summary Lecture 2:

- MQT and MQC: tunneling of collective dof
- Embedded in environment: dissipative tunneling
- Nanomagnet, SQUID, BEC
  
  ....vortices, Helium-3,4...

Thanks
Set-up

Lecture 1: Basics and Theoretical Tools

Lecture 2: Tunneling in Macroscopic Systems

Lecture 3: Tunneling in Josephson systems

- Josephson junctions
- Switching rates
- From classical activation to MQT
- Experimental results
- Tunneling of quantum bits

Goal: Understanding MQT and MQC in Josephson devices
Josephson-junction

Two wave functions overlap!

\[ \langle \psi | L \rangle = \sqrt{\rho} \ e^{i\varphi_L} \quad \langle \psi | R \rangle = \sqrt{\rho} \ e^{i\varphi_R} \]

\[ H = E_L \ | L \rangle \langle L | + E_R \ | R \rangle \langle R | + E_J \ (| L \rangle \langle R | + | R \rangle \langle L |) \]

\[ E_R - E_L = eV \]
Solve time dependent Schrödinger equation for $\rho$, $\varphi = \varphi_R - \varphi_L$

Josephson relations:

$$I = I_0 \sin(\varphi), \quad \dot{\varphi}(t) = V/\varphi_0$$

$$\varphi_0 = \hbar/2e$$
Electrical circuit
Josephson relations:

\[ I_S = I_0 \sin(\phi) \]
\[ V = \varphi_0 \frac{d\phi}{dt} \]

\[ I_b = I_R + I_S + I_C \]
\[ I_b = \frac{V}{R} + I_0 \sin(\phi) + CV \dot{V} \]
\[ I_b = \frac{\varphi_0}{R} \dot{\phi} + I_0 \sin(\phi) + \varphi_0 \ddot{\phi} \]

Motion of a classical particle in tilted periodic potential
Switching to finite voltage: Influence of thermal environment

\[ V(\varphi) \]

\[ I \]

\[ -2\Delta/e \]

\[ 2\Delta/e \]

\[ -I_0 \]

\[ I_0 \]
Switching to finite voltage: Influence of thermal environment

A voltage appears if $|I| > I_{th}$

Escape by thermal activation

Or

Macroscopic tunneling
Translational rules:
\[ \varphi_0^2 C \rightarrow M \]
\[ \frac{1}{RC} \rightarrow \gamma \]
\[ \langle I_b \rangle = \langle I_b \rangle + \delta I \]
\[ \langle I_b \rangle \varphi_0 \rightarrow F \]
\[ \varphi_0 \delta I \rightarrow \xi \]

Classical phase dynamics:
\[ M \ddot{\varphi}(t) + \gamma \dot{\varphi}(t) + V'(\varphi) = \xi(t) \]
\[ \langle \xi(t) \rangle = 0 \]
\[ \langle \xi(t) \xi(0) \rangle = 2M \gamma k_B T \delta(t) \]
\[ V(\varphi) = -E_J \cos(\varphi) - F \varphi \]
\[ E_J = I_0 \varphi_0 \]
Quantum mechanics

\[ [\hat{Q}, \hat{\varphi}] = i e \]

Macroscopic Quantum Tunneling

\[ V(\varphi) = -E_J \cos(\varphi) - F\varphi \]
Experimental protocol

Cool down: 20mK...100mK

Determine $E_J, E_C$ : $I_c$, \[ \omega = (\sqrt{2E_CE_J/\hbar})(1-s)^{1/4}, \quad s = I_b/I_0 \]

High temperature:

\[ \Gamma \propto e^{-\beta V_b} \]

\[ V_b \approx \frac{4\sqrt{2}}{3} E_J(1 - s)^{3/2} \]

\[ \ln[\omega_0/(2\pi\Gamma)]^{2/3} \]

$T = 400\text{ mK}$

$I_b(\mu A)$
Experimental protocol:

Cool down: 20mK...100mK

Determine 

\[ E_J, E_C : I_c, \omega = \sqrt{2E_CE_J/\hbar}(1-s)^{1/4}, s = I_b/I_0 \]

High temperature, external microwaves (power P): resonant activation

Enhancement

\[ \ln \Gamma(P)/\Gamma(0) \]

From the resonant enhancement:

Plasma frequency

Friction Q (width)
Experimental protocol:

Cool down: 20mK....100mK

Determine $I_c, \omega, Q$

Rise the bias current

Detect a switching via a voltage pulse

Repeat for many times

Make a histogram: #switchings vs bias current

Extract Tunneling rate
\[ \frac{\text{const}}{|\ln(\Gamma)|} \]

\[ T_{\text{esc}} \] vs \( T \) (mK)

- \( I_0 = 24.873 \pm 0.004 \, \mu\text{A} \)
- \( C = 4.28 \pm 0.34 \, \text{pF} \)
- \( R = 9.3 \pm 0.1 \, \Omega \)

Devoret et al, 1988
**Experiment**

**Thermal activation**

\[
\frac{\text{const}}{|\ln(\Gamma)|} \quad \text{T}_{\text{esc}} \quad (Q = \infty)
\]

\[
T_{\text{esc}} = \frac{4.28 \pm 0.34 \ \mu\text{A}}{24.873 \pm 0.004 \ \mu\text{A}}
\]

\[
R = 9.3 \pm 0.1 \ \Omega
\]

**Quantum tunneling**
Experiment

Enhancement due to quantum fluctuations above crossover

\[ \frac{\Gamma}{\Gamma_{cl}} \]

Enhancement due to quantum fluctuations for very low T

\[ \frac{\Gamma(T)}{\Gamma(T = 0)} \propto T^2 \]
Designed Qubit Systems

Exponential sensitivity to qubit state parameters
Superconducting Qubits

Rabi-oscillations in a two level atom

Artificial (mesoscopic) atom

Groupe Quantronique, CEA Saclay
Superconducting circuit: decoupling from excitations

States $|1\rangle$ and $|0\rangle$ related to supercurrents $i_1$ and $i_0$
Readout: Quantronium

\[ \delta \]

\[ MQT \]

\[ -E_j' \cos(\delta) - \delta I_{tot} \]

Quantronium (Charging regime)

Hamiltonian for \( N=\{0,1\} \) excess Cooper pairs:

\[
H = E_C \left( N_g - \frac{1}{2} \right) \sigma_z - E_J \cos \left( \frac{\delta + \phi}{2} \right) \sigma_x + \frac{p^2}{2m} - E'_J \cos(\delta) - \frac{I}{I_0} \delta
\]

Qubit

Read-out JJ

\[
N_g = C_g U / 2e \quad \text{Gate voltage}
\]

\[
\phi = \Phi / (\hbar / 2e) \quad \text{Magnetic flux}
\]

\[
N_g = \frac{1}{2}, \; \delta + \phi = \pi \quad \text{Crossing}
\]
Tunneling of a qubit: Crossing of surfaces

Flip: Smaller barrier $\rightarrow$ larger rate ?

Landau-Zener transitions "under" the barrier: MQT of a Spin

$H = \begin{pmatrix}
\frac{p^2}{2m} + V_+ & \Delta(\delta) \\
\Delta(\delta) & \frac{p^2}{2m} + V_-
\end{pmatrix}$

Vion et al & JA, PRL 94, 057004 (2005)
Spin-flip tunneling

Simple bounce

Flip bounce
MQT-Flip Rate

\[ \Gamma = \Gamma_0 + \Gamma_F \]

Non-flip \quad Flip

Prediction: Rate enhancement:

\[ \epsilon/j = 0.1 \]
\[ j = 0.05 \]
\[ i_b = 0.95 \]
\[ V_r/\omega = 4 \]

JA et al., PRL 016803 (2003)
Experiment: Rate Enhancement

Theory vs. Experiment

Ithier et al, PRL 94, 057004 (2005)
Summary Lecture 3:

- Josephson junction: fictitious particle
- Switching rate measurements, crossover
- Spin-flip tunneling in Josephson qubits

Thanks