Departments of Physics
and Applied Physics,
Yale University

Quantum Optics with Electrical Circuits:
Strong-coupling Circuit QED

Jens Koch

KECK FOUNDATION
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Departments of Physics
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Circuit QED at Yale

EXPERIMENT

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Jerry Chow
Joseph Schreier
Blake Johnson
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Joe Chen (Cornell)
Cliff Cheung (Harvard)
Aashish Clerk (McGill)
R. Huang (Taipei)
K. Moon (Yonsei)

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Appetizer – a flavor of what's to come

Quantum Optics with Electrical Circuits

- optics beyond classical electrodynamics
- single photons

- **quantum** electrical circuits!
- How to quantize an electrical circuit?

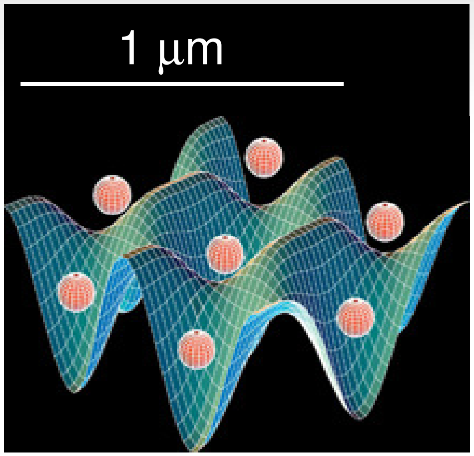
Strong-coupling Circuit QED

- qubits and photons interact strongly
  - ► "phobits" and "qutons"
  - ► vacuum Rabi effect

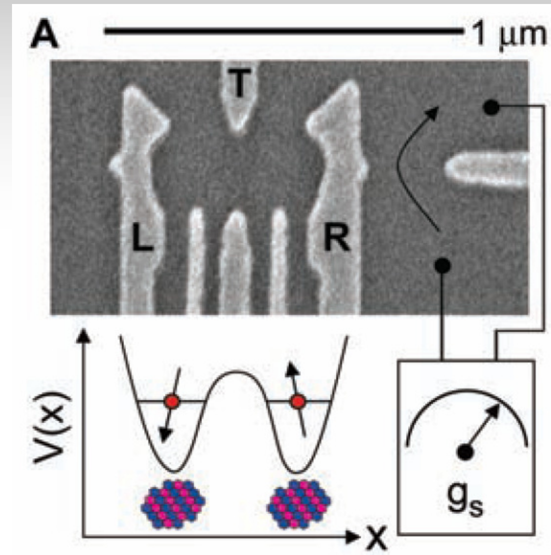
- coupling artificial atoms ("qubits") to on-chip radiation field
- exciting applications in quantum computing and optics

Merger of AMO and CM physics

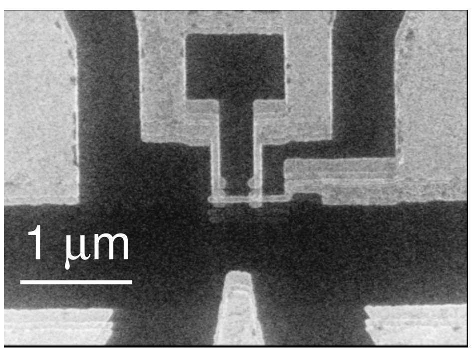
- Atoms and lasers ◄ ► Many-body physics



  - many microscopic d.o.f.
  - tunable interactions
  - switch lattice on/off
  - long coherence times
  - readout by optical imaging



- Nanofab and electronics ◄ ► Quantum optics



  - macroscopic d.o.f.
  - tunable Hamiltonian
  - modest coherence times
  - electrical readout

Non-linear elements:
Quantum Dots
Josephson Junctions

Recently: Cavity QED with a BEC!

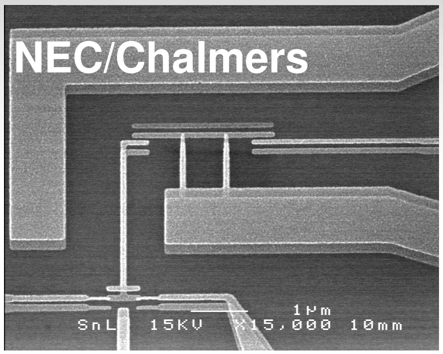
Brennecke et al., arXiv:0706.3411v1 [quant-ph]

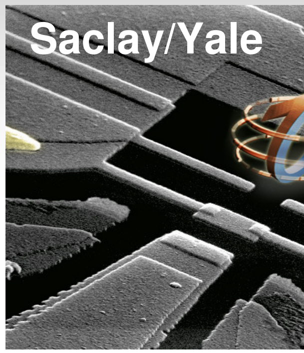
Superconducting Qubits

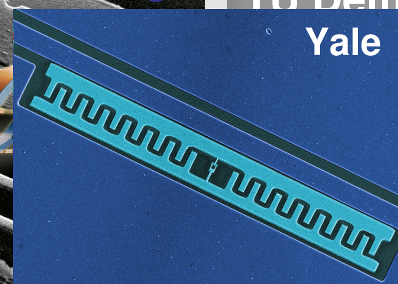
Nonlinearity from Josephson junctions (Al/AlO_x/Al)

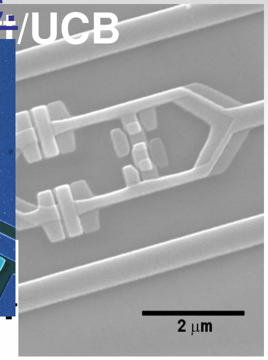
Charge	Charge/Phase	Flux	Phase

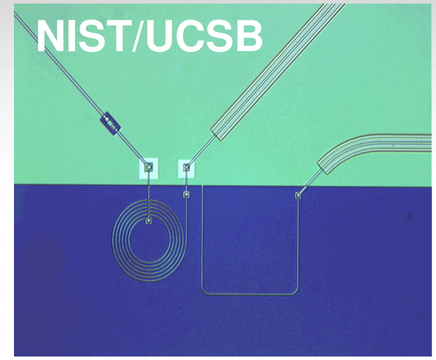
Charge/ac Susc.











$E_J = 30\text{-}100E_C$

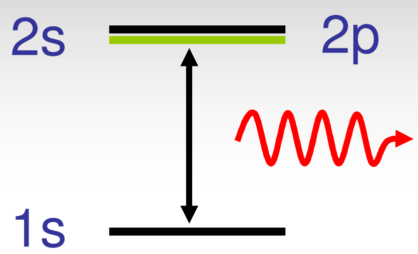
$E_J = E_C$	$E_J = E_C$	$E_J = 40\text{-}100E_C$	$E_J = 10{,}000E_C$

- 1st superconducting qubit operated in 1998 (NEC Labs, Japan)
- "long" coherence shown 2002 (Saclay)
- two examples of C-NOT gates (2003, NEC; 2007, Delft and UCSB)
- Bell inequality tests being attempted (2006, UCSB)

So far mostly classical E-M fields: atomic physics with circuits

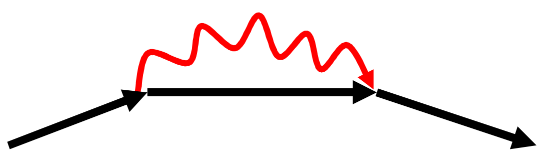
Goal: interaction w/ ***quantized*** fields → Quantum optics with circuits

QED: Atoms Coupled to Photons



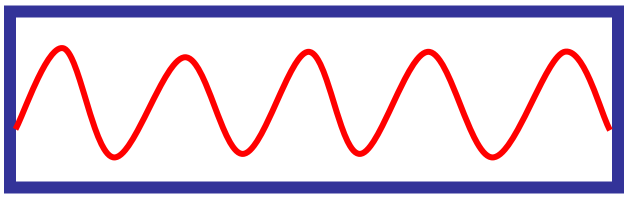
Irreversible spontaneous decay into the photon continuum:

$$2p \rightarrow 1s + \gamma \qquad T_1 \sim 1\mathrm{ns}$$



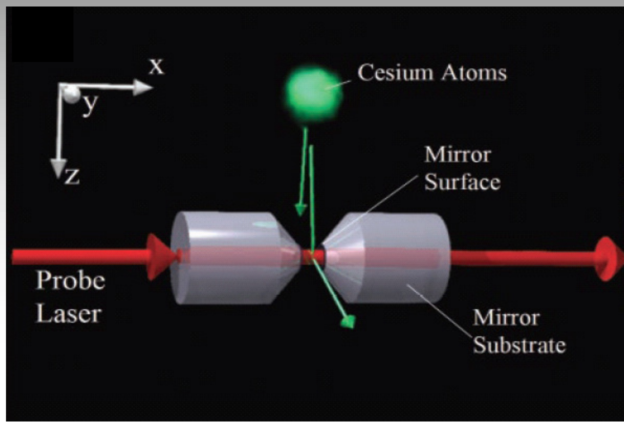
Vacuum Fluctuations:
(virtual photon emission and reabsorption)

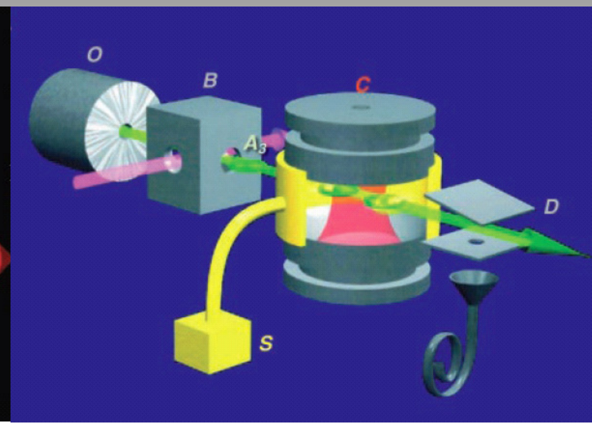
Lamb shift lifts 2s - 2p degeneracy

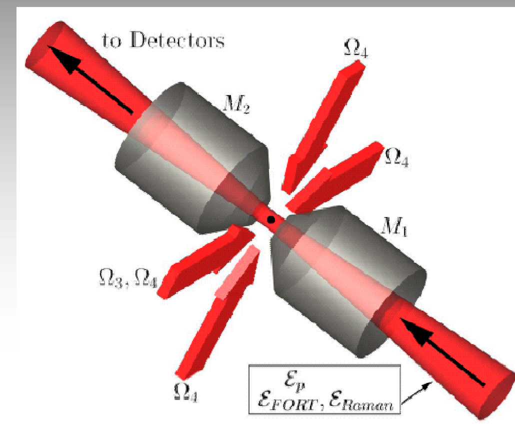


Cavity QED:
What if we trap photons as discrete modes inside cavity?

Strong-coupling cQED



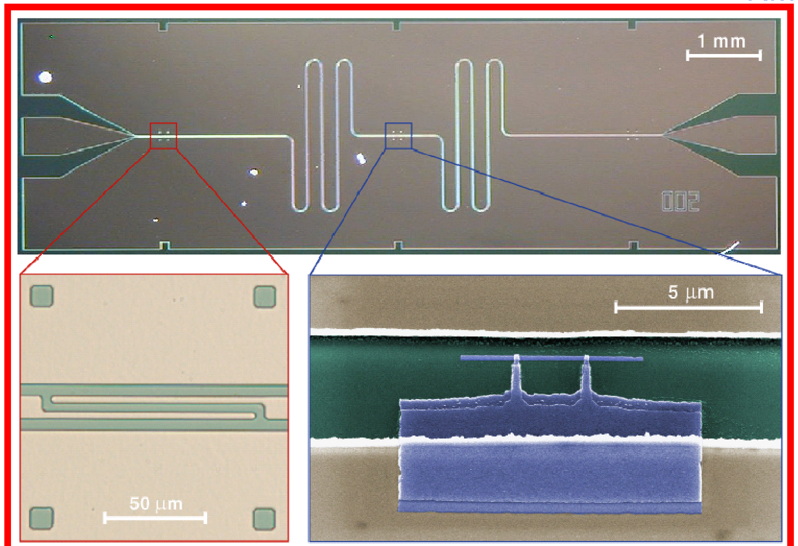




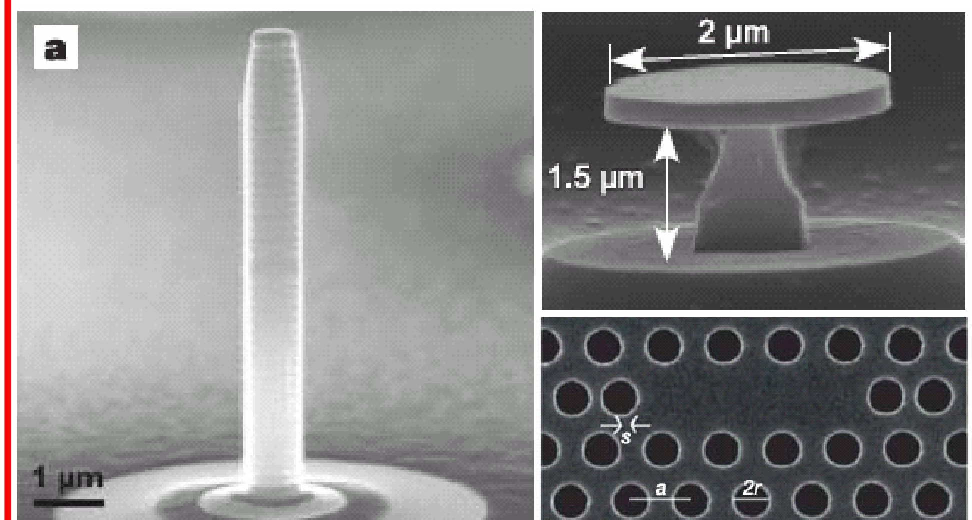
alkali atoms
Science **287**, 1447 (2000)

Rydberg atoms
RMP **73**, 565 (Dec. 2001)

single trapped atom
PRL **93**, 233603 (Dec. 2004)



superconducting flux and charge qubits
Nature (London) **431**, 159 (Sept. 2004)



semiconductor quantum dots
Nature (London) **432**, 197 (2004); ibid. **432**, 200 (2004)

Lecture 1: Introduction to cavity QED and superconducting qubits

1 • Intro to cavity QED (cQED)

2 • superconductivity for pedestrians
• What is a Josephson junction?

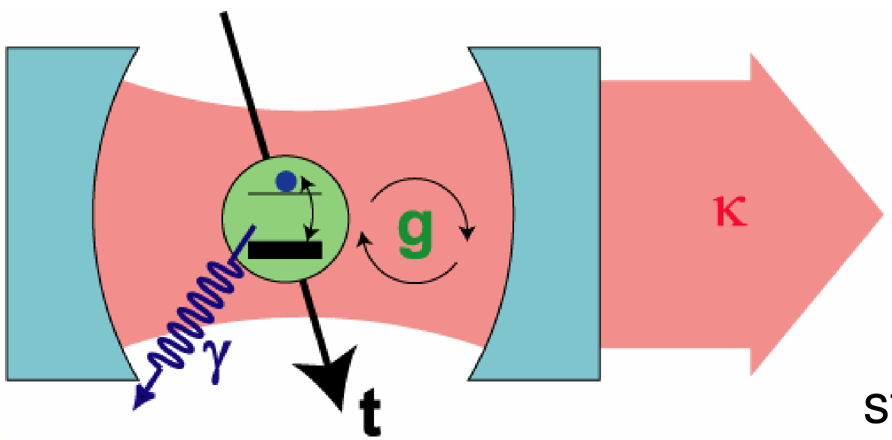
3 • Cooper pair box qubit I
circuit quantization

Cavity Quantum Electrodynamics

What is cQED?

- coupling atom / discrete mode(s) of EM field
- central paradigm for study of open quantum systems

▶ coherent control, ▶ quantum information processing
▶ conditional quantum evolution, ▶ quantum feedback
▶ decoherence



2g = vacuum Rabi freq.

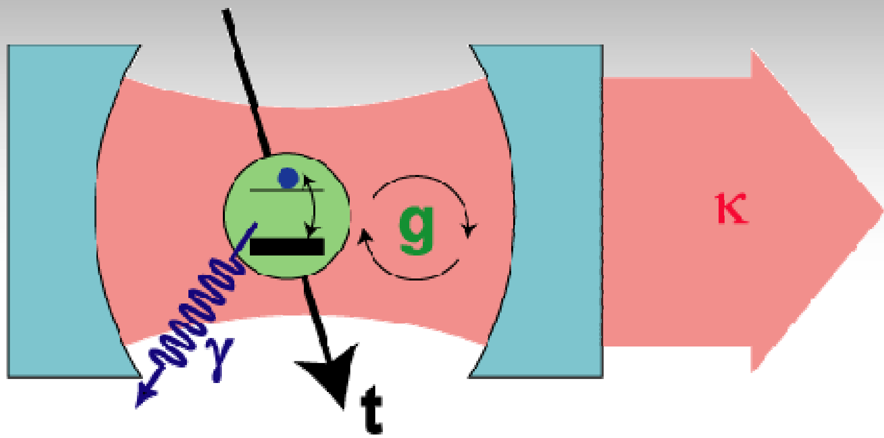
κ = cavity decay rate

γ = "transverse" decay rate

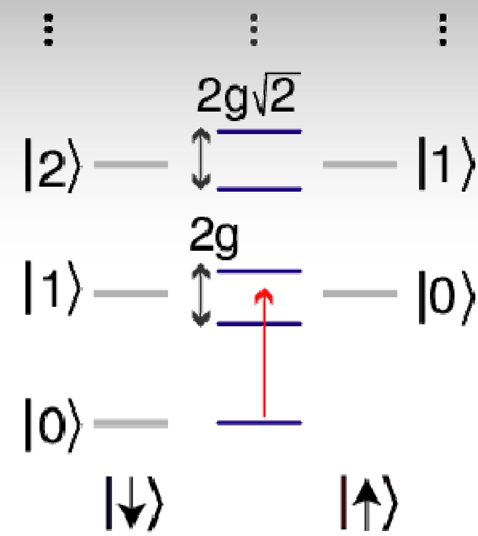
t = transit time

strong coupling: $g > \kappa, \gamma, 1/t$

Cavity Quantum Electrodynamics



on resonance:
$(\omega_r = \omega_a)$



Jaynes-Cummings Hamiltonian

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a})$$

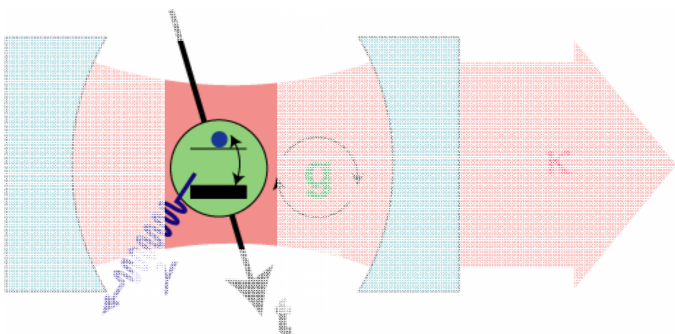
quantized field

2-level system

atom-photon interaction

Cavity QED

What is the 2-level system?
How do we describe it?



$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a})$$

quantized field

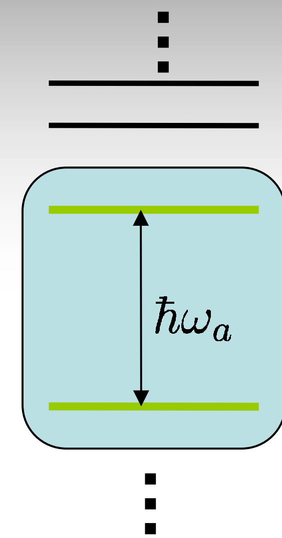
2-level system

electric dipole interaction

Atoms for 2-level systems

Requirements:

- anharmonicity (natural!)
- long-lived states
- good coupling to EM field
- preparation, trapping etc.



Rydberg atoms & microwave cavities (Haroche et al.)

Excited atoms with one (or several) e^- in very high principal quantum number (n)

► long radiative decay time ($\sim 3 \times 10^{-2}\mathrm{s}$),

► very large dipole moments

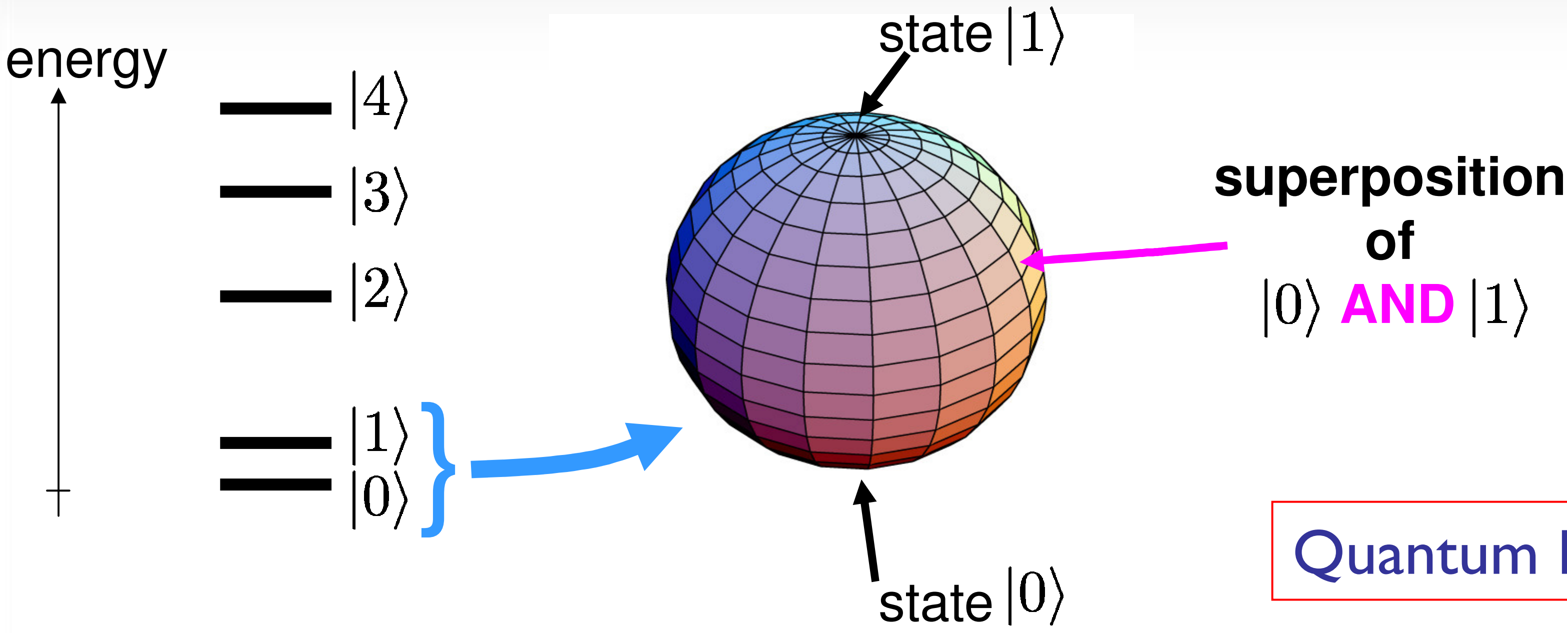
► well-defined preparation procedure

Alkali atoms trapped in optical cavities (Kimble et al.)

can trap single atom inside optical cavity,
manipulate and read out its state with lasers!

Quantum Bits and Information

2-level quantum system (two distinct states $|0\rangle, |1\rangle$)
can exist in an **infinite number**
of physical states *intermediate* between $|0\rangle$ and $|1\rangle$.



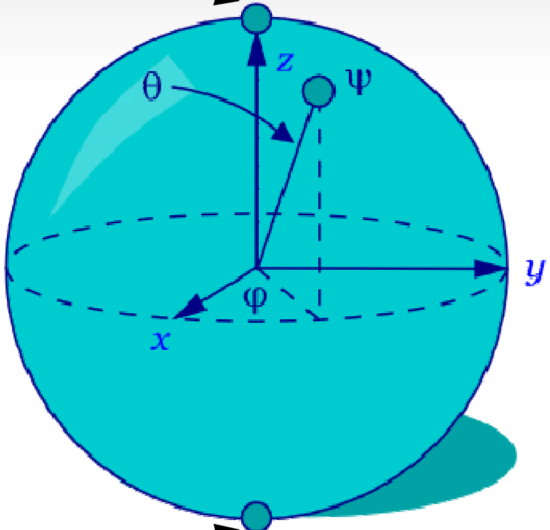
Quantum Bit = qubit

System can be in ‘both states at once’
just as it can take two ***paths*** at once.

Bloch sphere, qubit superpositions

Bloch sphere: geometric representation of qubit states as points on the surface of a unit sphere

state $|1\rangle$



state $|0\rangle$

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$$

ignoring global phase factor

$$|\Psi\rangle = \cos\tfrac{\theta}{2}|0\rangle + e^{i\varphi}\sin\tfrac{\theta}{2}|1\rangle$$

equiv. $|\Psi\rangle = e^{-i\varphi/2}\cos\tfrac{\theta}{2}|0\rangle + e^{i\varphi/2}\sin\tfrac{\theta}{2}|1\rangle$

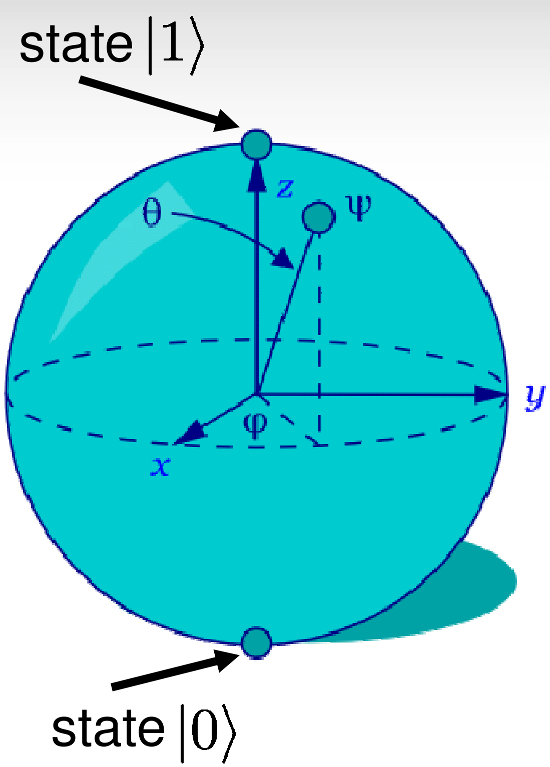
Latitude and longitude on the 'Bloch sphere'

$$0 \leq \theta \leq \pi, \quad 0 \leq \varphi < 2\pi$$

Any superposition state: represented by arrow (called 'spin') pointing to a location on the sphere

nice discussion: http://www.vcpc.univie.ac.at/~ian/hotlist/qc/talks/bloch-sphere.pdf

Bloch sphere and density matrix



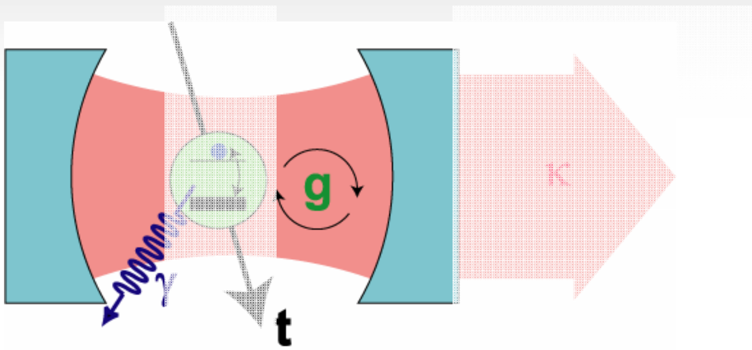
As an aside:

Can generalize this to density matrices!
Whole interior of the sphere has meaning.

$$\rho = \frac{1}{2}(\mathbb{1} + r_x \hat{\sigma}^x + r_y \hat{\sigma}^y + r_z \hat{\sigma}^z)$$

$$\mathbf{r} = (r_x, r_y, r_z), \quad |\mathbf{r}|^2 \leq 1$$

Cavity QED



the quantized EM field

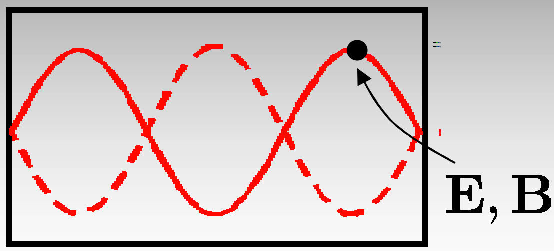
$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger\hat{\sigma}^- + \hat{\sigma}^+\hat{a})$$

quantized field

2-level system

electric dipole interaction

Quantizing the EM Field: Photons

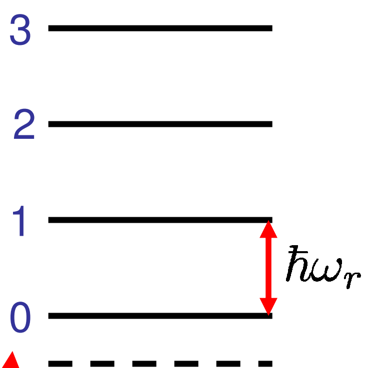


cavity volume: V_c

$$\hat{H} = \int_{V_c} d^3r \left[\frac{1}{2}\epsilon_0 \hat{\mathbf{E}}^2(\mathbf{r},t) + \frac{1}{2\mu_0}\hat{\mathbf{B}}^2(\mathbf{r},t) \right]$$

$\hat{x} \leftrightarrow \hat{E}$ $\qquad$ $\hat{p} \leftrightarrow \hat{B}$

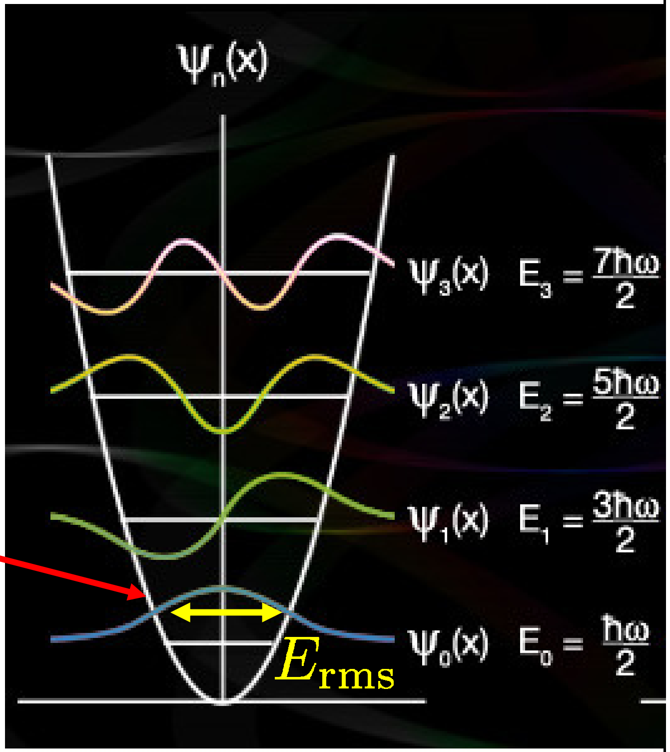
Quantization of radiation field:
Collection of harmonic oscillators!



photon number

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_r$$

Zero-point
'vacuum fluctuation'
energy



see, e.g., S.M. Dutra, *Cavity Quantum Electrodynamics* (Wiley 2005)

Vacuum fluctuations of E field

$$E_{\rm rms} = \sqrt{\langle 0|\hat{\mathbf{E}}^2|0\rangle}$$

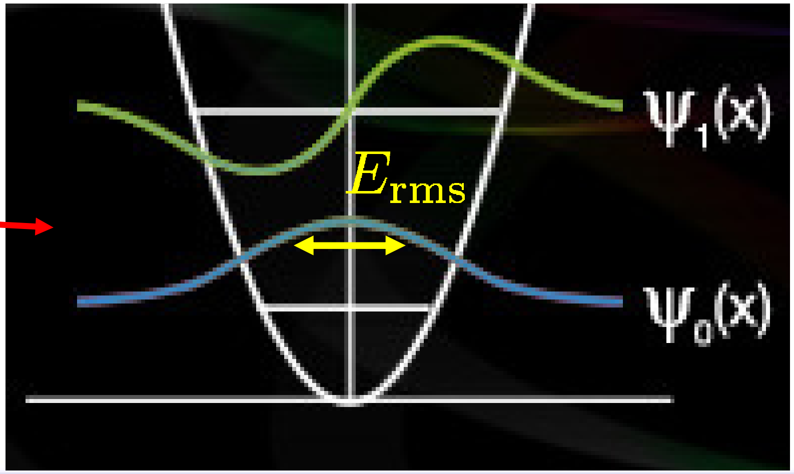
need: $\hat{E} \leftrightarrow \hat{a},\ \hat{a}^\dagger$

mnemonic trick: $$V_{\rm c}\left[\frac{1}{2}\epsilon_0\langle\hat{\mathbf{E}}^2\rangle\right] = \frac{1}{2}\left[\frac{1}{2}\hbar\omega_r\right]$$

$$\Rightarrow \quad E_{\rm rms} = \sqrt{\langle\hat{\mathbf{E}}^2\rangle} = \sqrt{\frac{\hbar\omega_r}{2\epsilon_0 V_{\rm c}}}$$

small cavity enhances quantum fluctuations of electric field!

Zero-point vacuum fluctuations



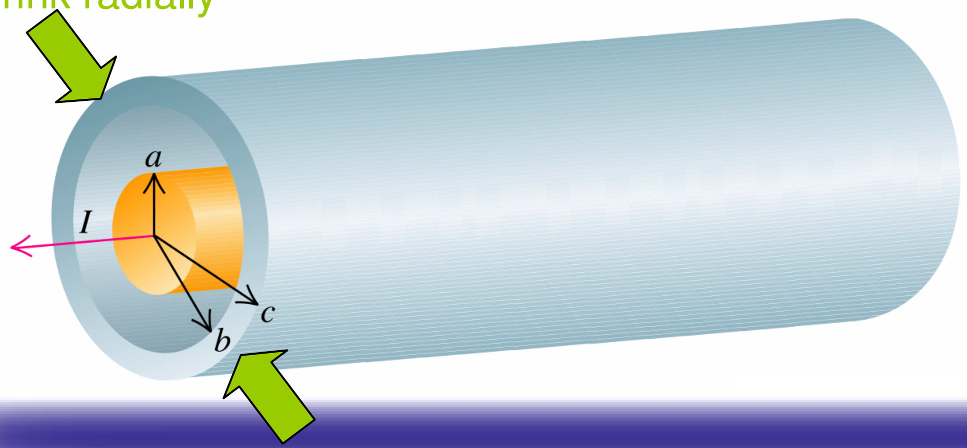
Vacuum fluctuations of E field II

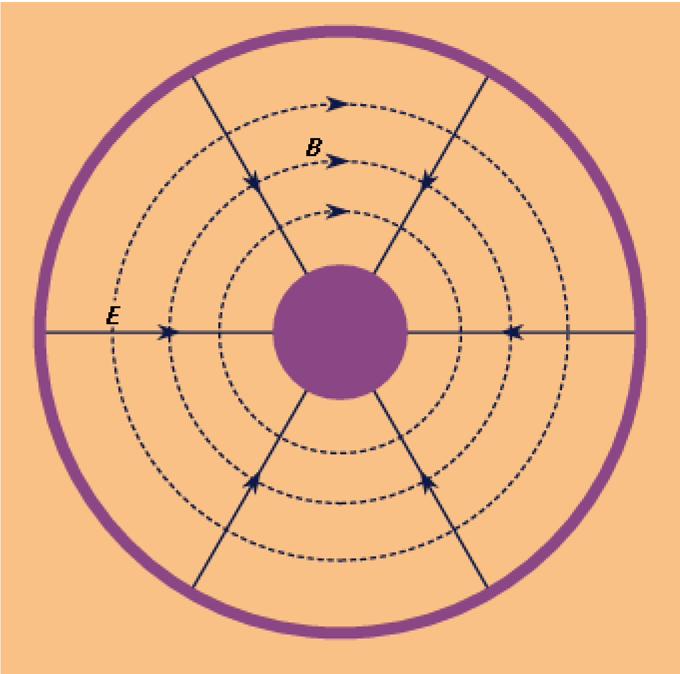
$$E_{\rm rms} = \sqrt{\frac{\hbar\omega_r}{2\epsilon_0 V_c}} \overset{?}{\sim} \frac{1}{\sqrt{V_c}}$$

Caveat: In general, ω_r changes as well!
In special cases, ω_r can be held constant.

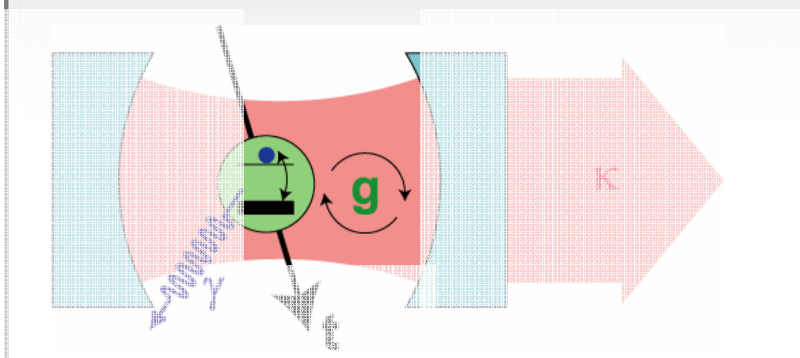
<u>Example:</u> Cavity formed by long coax cable

shrink radially





Cavity QED



the coupling

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger\hat{\sigma}_- + \hat{\sigma}_+\hat{a})$$

quantized field

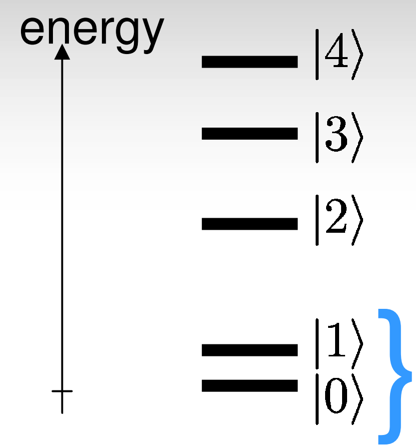
2-level system

electric dipole interaction

Coupling between atom and EM field

Electric dipole moment couples to electric field!

$$E_{\text{dipole}} = -\mathbf{d}\cdot\mathbf{E} \rightarrow -q\,\hat{x}\,E_{\text{rms}}(\hat{a}+\hat{a}^\dagger)$$

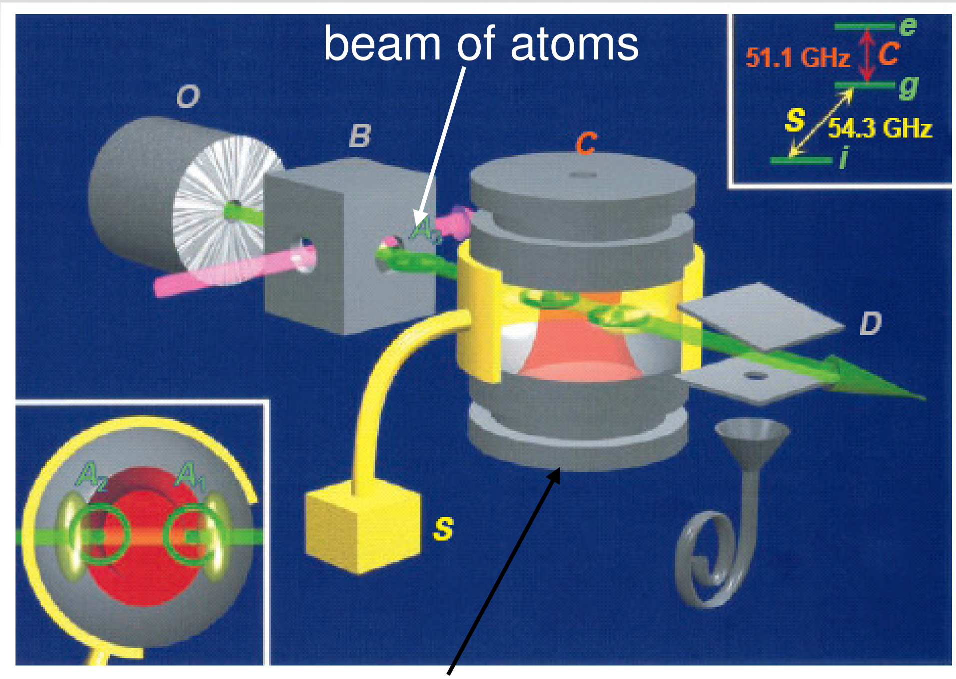


$$\sum_{i=0,1}\sum_{j=0,1} |i\rangle \underbrace{\langle i|q\,\hat{x}|j\rangle}_{d_{ij}} \langle j|$$

$$d_{01}(\hat{\sigma}_+ + \hat{\sigma}_-)$$

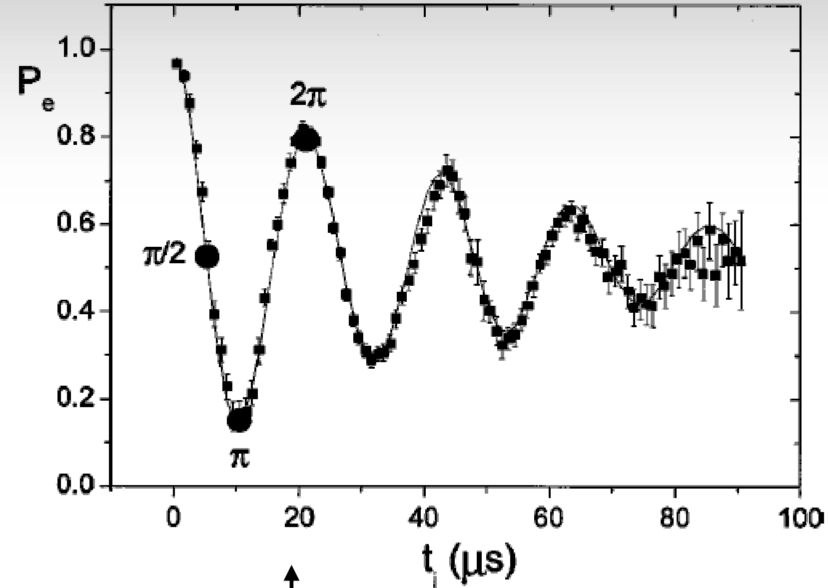
$$\hat{H}_{\text{coupling}} = \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-)(\hat{a}+\hat{a}^\dagger) \xrightarrow{\text{RWA}} \hbar g(\hat{\sigma}_+\hat{a} + \hat{a}^\dagger\hat{\sigma}_-)$$

μwave cQED with Rydberg Atoms



3-d superconducting cavity
(50 GHz)

vacuum Rabi oscillations

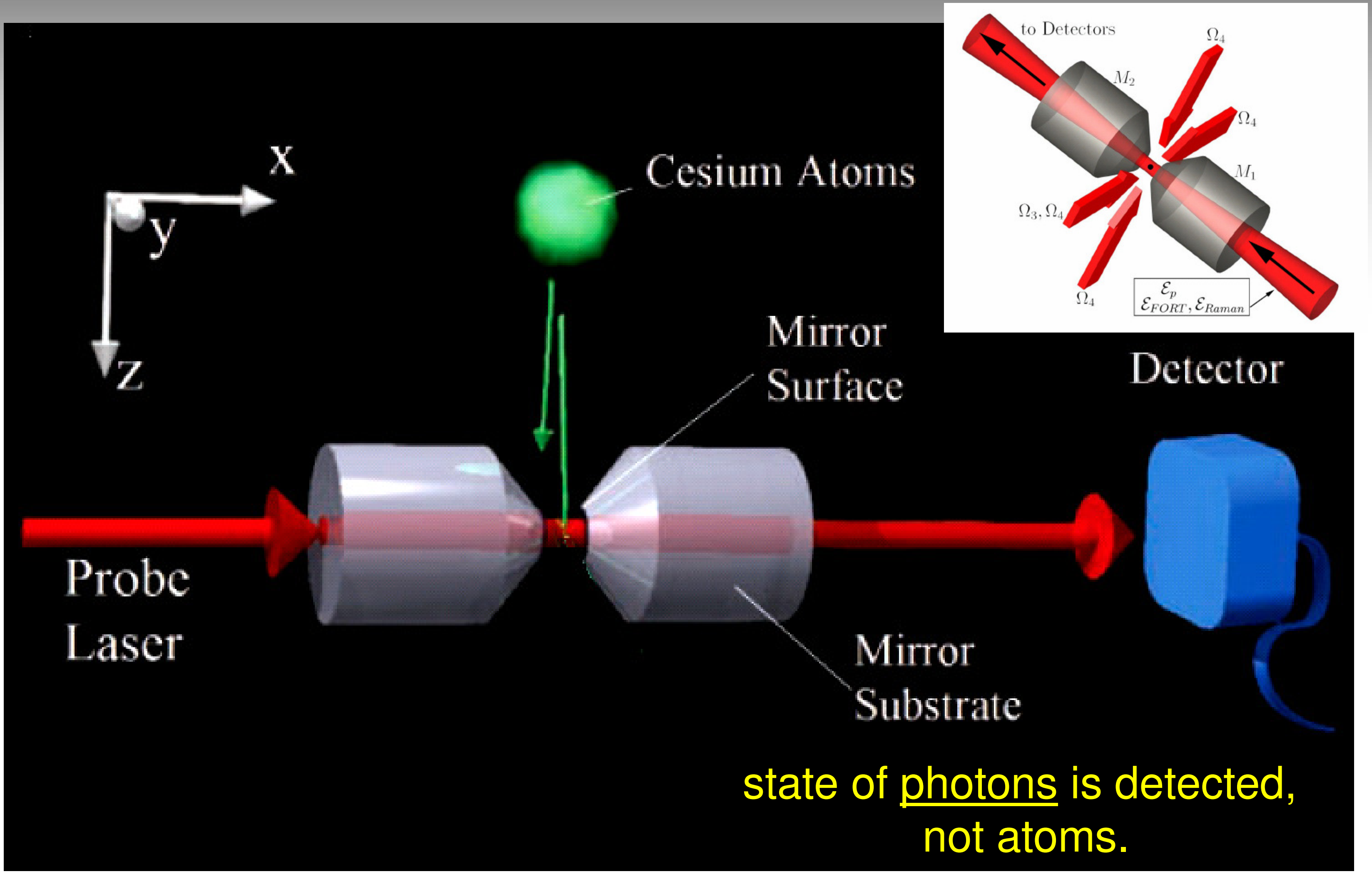


observe dependence of atom final state on time spent in cavity

measure atomic state, or …

Review: S. Haroche et al., Rev. Mod. Phys. **73** 565 (2001)

cQED at optical frequencies



… measure changes in transmission of optical cavity

(Caltech group H. J. Kimble, H. Mabuchi)

Superconductivity, Josephson junctions

What is superconductivity?

[happy 50th birthday, BCS!]

vanishing dc resistivity

Meissner effect

signature in heat capacity (phase transition!)

isotope effect e-ph coupling!

Fermi sea unstable for attractive e^--e^- interaction

e^- pairing in k space Cooper pairs

Cooper pairs form coherent state (BCS)

...

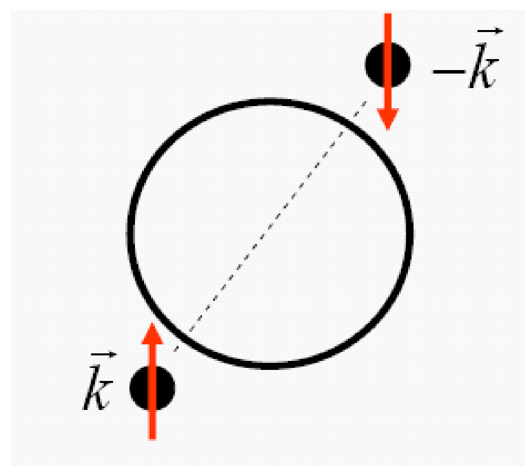
Complex order parameter (like BEC)

SC gap

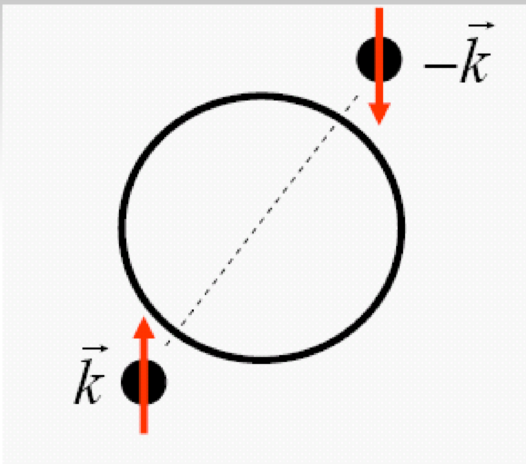
$$\psi \sim \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \sim \Delta$$

$$\psi = |\psi| e^{i\varphi}$$

SC phase

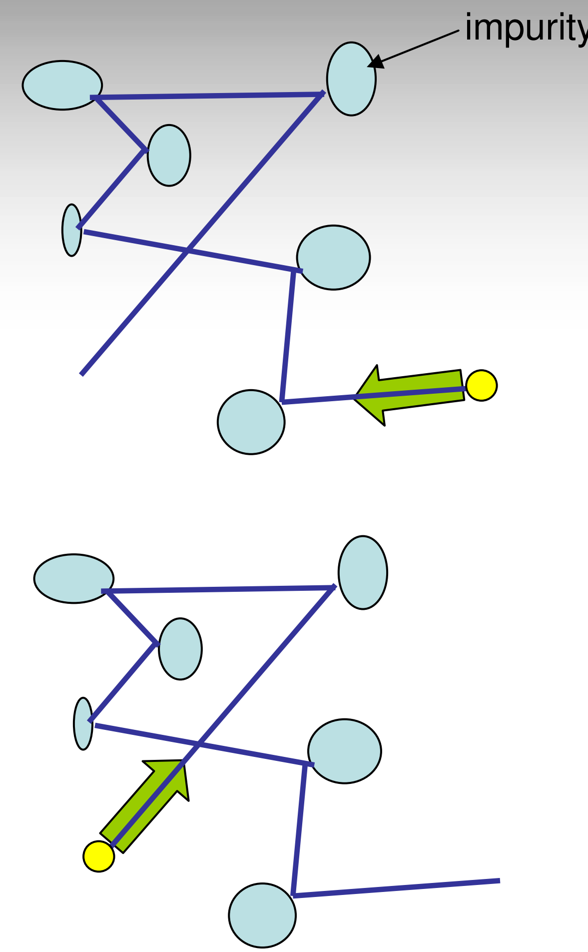


What is superconductivity?



clean crystal
momentum space

general case:
coupling of
time-reversed states

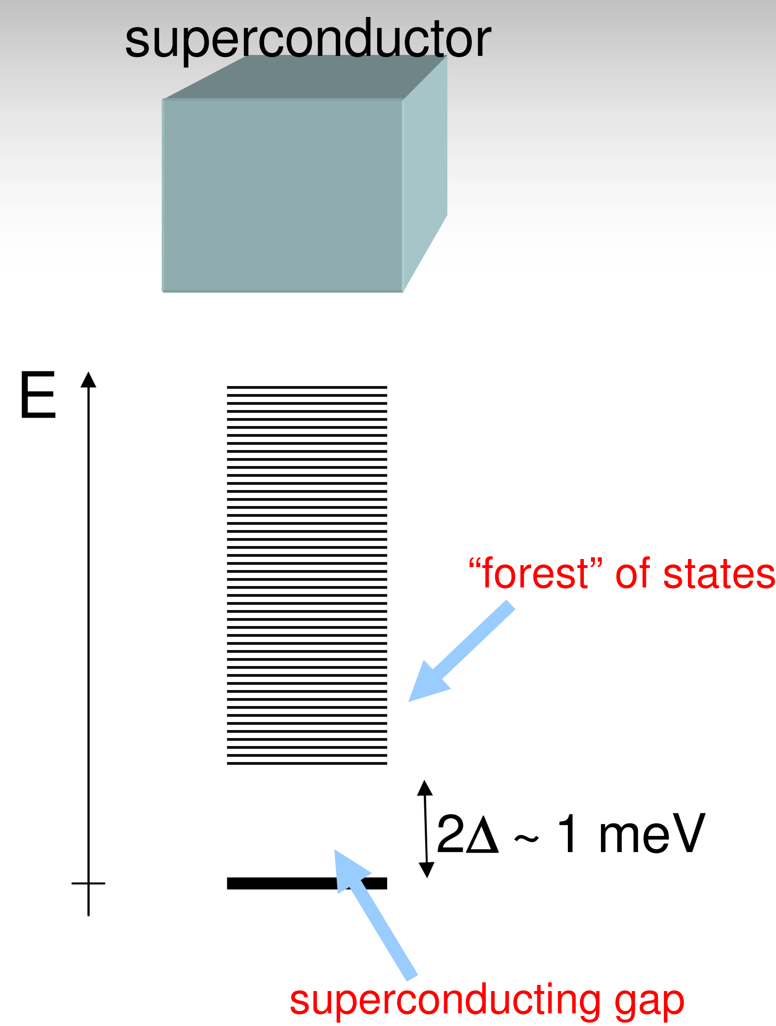


► can use dirty materials
for superconductors!

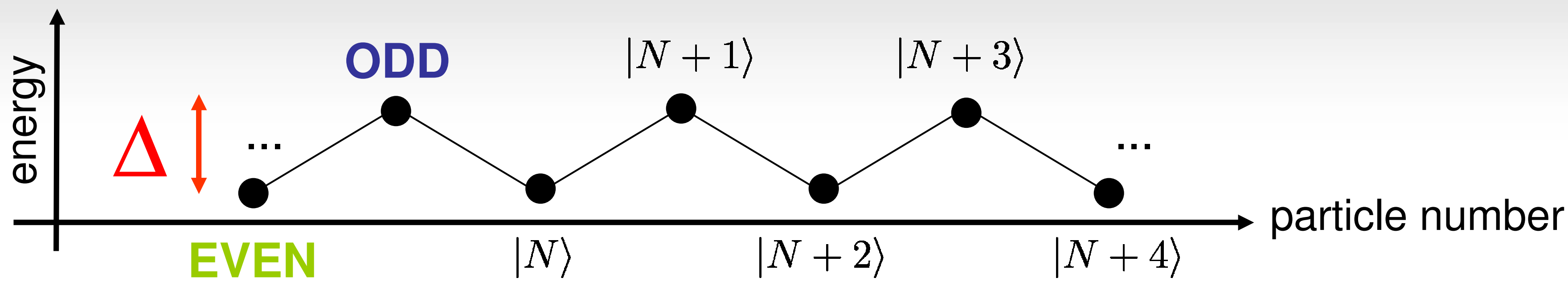
Why superconductivity?

Wanted:
► electrical circuit as artificial atom
► atom should not spontaneously lose energy
► anharmonic spectrum

Superconductor
► dissipationless!
► provides nonlinearity via Josephson effect



Energy vs. particle number on grain



(ignoring electrostatic charging energy)

•macroscopic SC: $N \sim 10^{23}$

► oftentimes, the question whether N is even or odd can be ignored

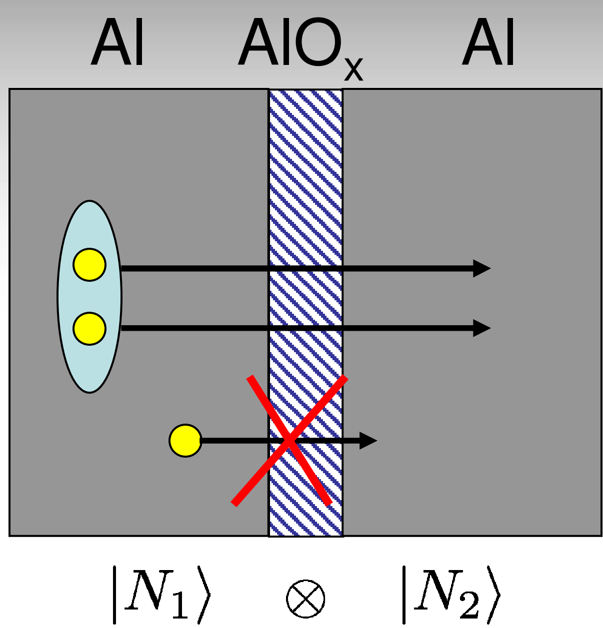
Tunneling between two SCs

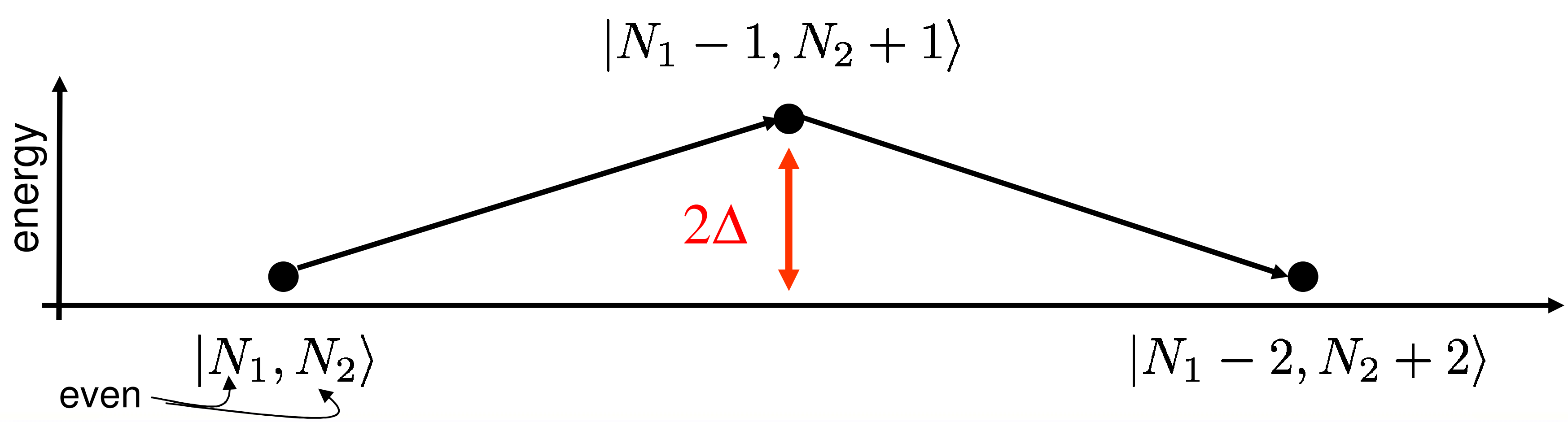
- couple two superconductors
  via oxide layer

- oxide layer acts as tunneling barrier

- superconducting gap inhibits e^- tunneling
  Cooper pairs CAN tunnel!

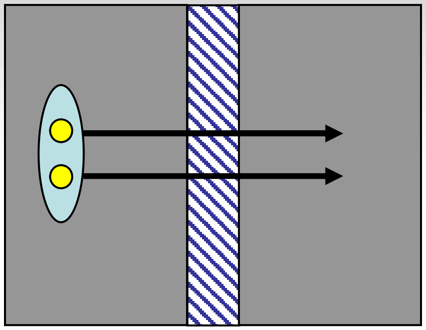
► **Josephson tunneling**
(2nd order with virtual intermediate state)





Josephson Tunneling

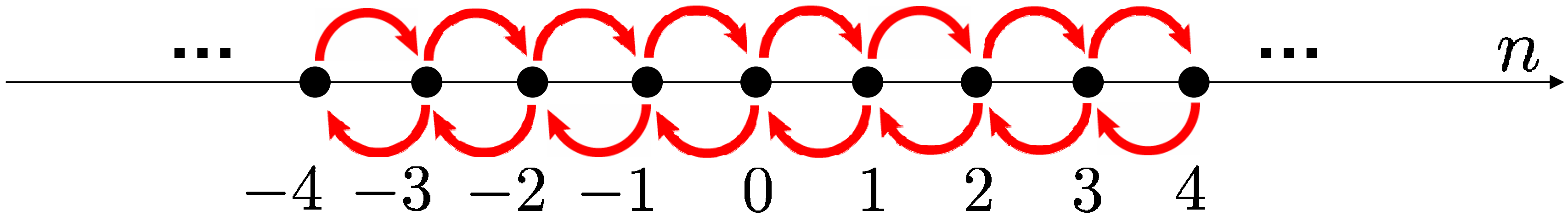
Coherent tunneling of Cooper pairs

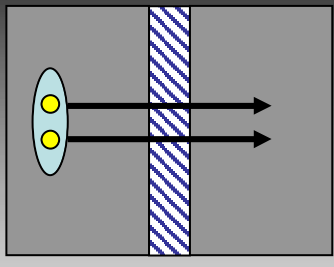


Restrict Hilbert space to quantum ground states with even e$^-$ numbers:

$$|N_1 - 2n, N_2 + 2n\rangle, \qquad n = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

(# of Cooper pairs that have tunneled from left to right)

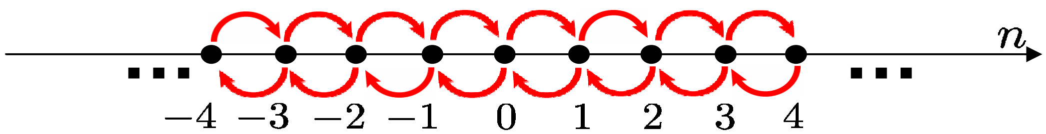




Josephson Tunneling I

Characterize basis states by **number of Cooper pairs** that have tunneled:

$$|n\rangle := |N_1 - 2n, N_2 + 2n\rangle, \quad n \in \mathbb{Z}$$



Tunneling operator for Cooper pairs:

$$\hat{H}_T = -\frac{E_J}{2} \sum_{n=-\infty}^{\infty} \Big[|n+1\rangle\langle n| + |n\rangle\langle n+1| \Big]$$

normal state conductance

SC gap

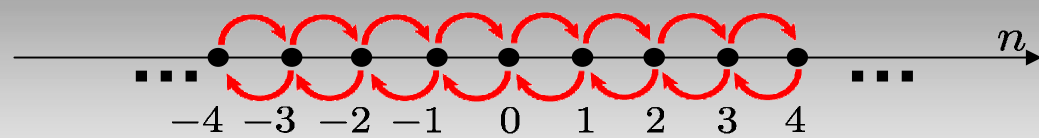
$$E_J = \frac{G_t \Delta}{8e^2/h}$$

Josephson energy

Tight binding model: hopping on a 1D lattice!

Note: $E_J \sim \Delta$ **NOT** $\cancel{E_J \sim 1/\Delta}$

Josephson Tunneling II



Tight binding model: $$\hat{H}_T = -\frac{E_J}{2} \sum_{n=-\infty}^{\infty} \Big[|n+1\rangle\langle n| + |n\rangle\langle n+1| \Big]$$

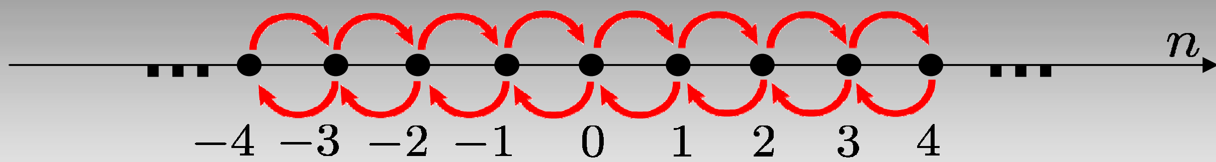
Diagonalization:

'position' $x_j \leftrightarrow n$

'wave vector' $k \leftrightarrow \varphi$ (compact!)

'plane wave eigenstate' $$|\varphi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{i\varphi n} |n\rangle \quad \leftrightarrow \quad \frac{1}{\sqrt{V}} \sum_j e^{ikx_j} |x_j\rangle$$

Josephson Tunneling III

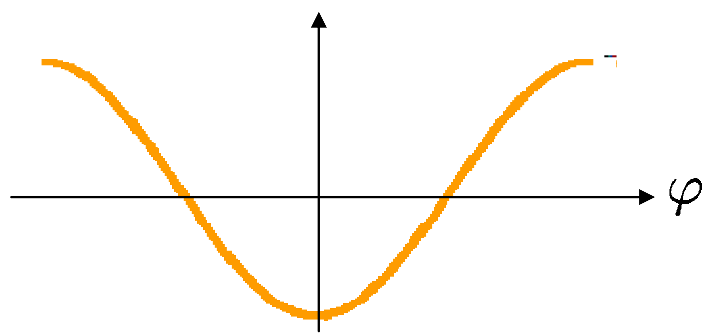


$$\hat{H}_T|\varphi\rangle = -\frac{E_J}{2}\sum_{n'=-\infty}^{\infty}\Big[|n'+1\rangle\langle n'| + |n'\rangle\langle n'+1|\Big]\frac{1}{\sqrt{2}}\sum_{n=-\infty}^{\infty} e^{in\varphi}|n\rangle$$

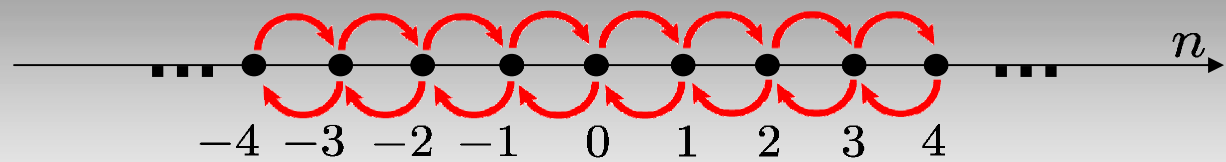
$$= -\frac{E_J}{2}\frac{1}{\sqrt{2}}\sum_{n=-\infty}^{\infty} e^{i\varphi n}\Big[|n+1\rangle + |n-1\rangle\Big]$$

$$= -\frac{E_J}{2}\frac{1}{\sqrt{2}}\Big[e^{-i\varphi}\sum_{n=-\infty}^{\infty} e^{i\varphi(n+1)}|n+1\rangle + e^{i\varphi}\sum_{n=-\infty}^{\infty} e^{i\varphi(n-1)}|n-1\rangle\Big]$$

$$= -E_J\cos\varphi|\varphi\rangle$$



Supercurrent through a JJ



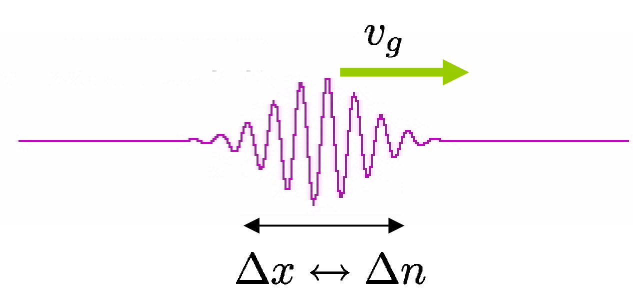
‘position’ $x_j \leftrightarrow n$

‘wave vector’ $k \leftrightarrow \varphi$

$$\hat{H}_T|\varphi\rangle = -E_J \cos\varphi|\varphi\rangle$$

Wave packet group velocity

$$\frac{dx}{dt} = v_g = \frac{d\omega}{dk} \quad \leftrightarrow \quad \frac{dn}{dt} = \frac{1}{\hbar}\frac{dH_T}{d\varphi} = \frac{E_J}{\hbar}\sin\varphi$$

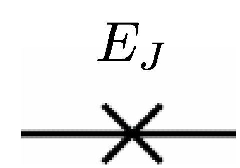


current: $$I = (2e)\frac{dn}{dt} = \frac{2e}{\hbar}E_J \sin\varphi$$

$$= I_c \sin\varphi$$

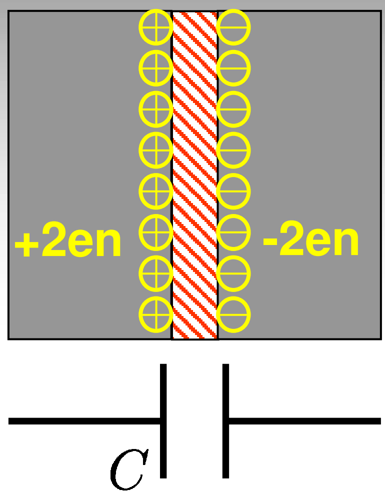
critical current $I_c = \frac{2e}{\hbar}E_J$

Josephson equation: current-phase relation



► the only non-linear non-dissipative circuit element!

Charging Energy



Transfer of Cooper pairs across junction

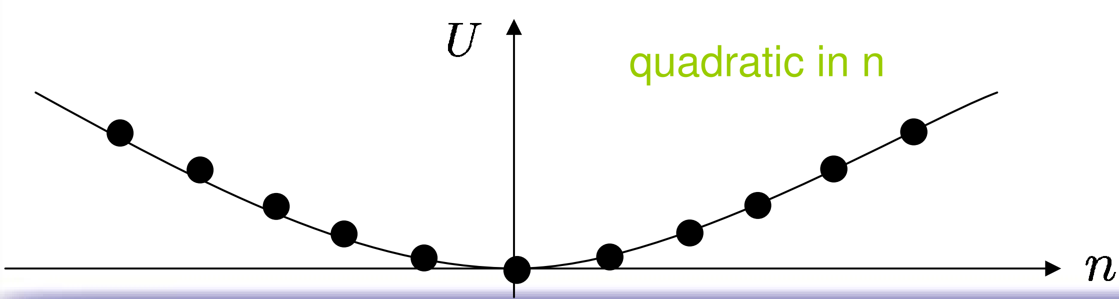
$$|N_1 - 2n, N_2 + 2n\rangle = |n\rangle, \quad n \in \mathbb{Z}$$

charging of SCs

► junction also acts as **capacitor**!

$$U = \frac{Q^2}{2C} = \frac{(2e)^2}{2C} n^2 \quad \Rightarrow \quad \hat{H}_U = 4E_c \hat{n}^2$$

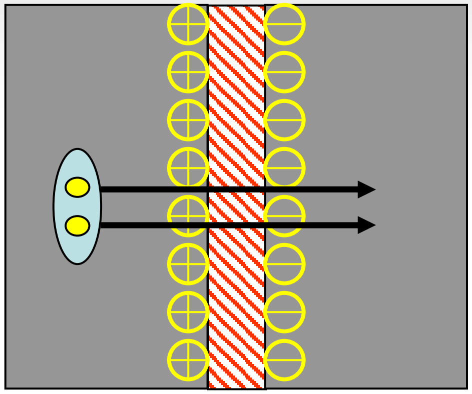
with $E_c = \dfrac{e^2}{2C}$

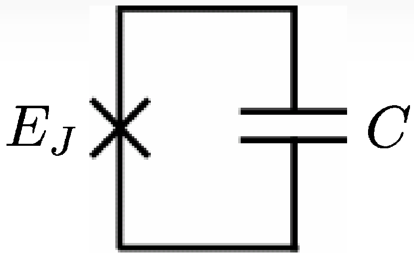


<u>charging energy</u>

Josephson tunneling + charging: the Cooper pair box

Combine Josephson tunneling and charging:





the **Cooper pair box** (CPB) Hamiltonian

$$\hat{H}_{\mathrm{CPB}} = \hat{H}_U + \hat{H}_T$$
$$= 4E_C\hat{n}^2 - \frac{E_J}{2}\sum_{n=-\infty}^{\infty}\Big[|n+1\rangle\langle n| + |n\rangle\langle n+1|\Big]$$

crucial parameter: E_J/E_C

Charge basis and phase basis

$$\hat{H}_{\mathrm{CPB}} = \hat{H}_U + \hat{H}_T$$

diagonal in charge basis:

$$\hat{H}_U|n\rangle = 4E_C n^2 |n\rangle$$

diagonal in phase basis:

$$\hat{H}_T|\varphi\rangle = -E_J \cos\varphi|\varphi\rangle$$

recall: $$|\varphi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{i\varphi n}|n\rangle$$

<u>orthonormality and completeness</u>

charge basis:

$$\langle n|n'\rangle = \delta_{nn'}$$

$$\sum_{n=-\infty}^{\infty} |n\rangle\langle n| = \mathbb{1}$$

phase basis:

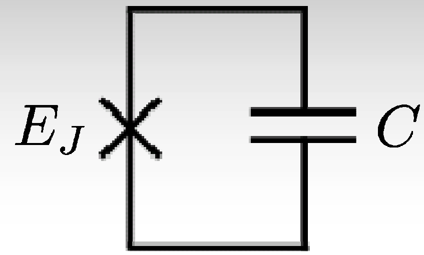
$$\langle\varphi|\varphi'\rangle = \delta(\varphi - \varphi')$$

$$\int_0^{2\pi} d\varphi|\varphi\rangle\langle\varphi| = \mathbb{1}$$

CPB Hamiltonian

in charge and phase basis

$$\hat{H}_{\mathrm{CPB}}|\Psi\rangle = E|\Psi\rangle$$



projections: $\Phi(n) = \langle n|\Psi\rangle,\quad \Psi(\varphi) = \langle\varphi|\Psi\rangle$

probability amplitude for number

probability amplitude for phase

charge basis:

$$4E_c n^2 \Phi(n) + \frac{E_J}{2}\Big[\Phi(n+1) + \Phi(n-1)\Big] = E\Phi(n)$$

$$\begin{pmatrix} \ddots & & & & \\ -\frac{E_J}{2} & 4E_c(-1)^2 & -\frac{E_J}{2} & & \\ & -\frac{E_J}{2} & 4E_c(0)^2 & -\frac{E_J}{2} & \\ & & -\frac{E_J}{2} & 4(+1)^2 & -\frac{E_J}{2} \\ & & & & \ddots \end{pmatrix}$$

numerical diagonalization

phase basis: $\hat{n} \rightarrow i\frac{d}{d\varphi}$

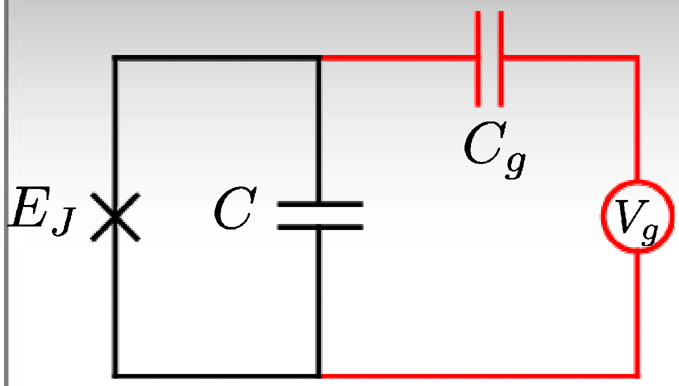
$$\Big[4E_c(i\frac{d}{d\varphi})^2 - E_J\cos\varphi\Big]\Psi(\varphi) = E\Psi(\varphi)$$

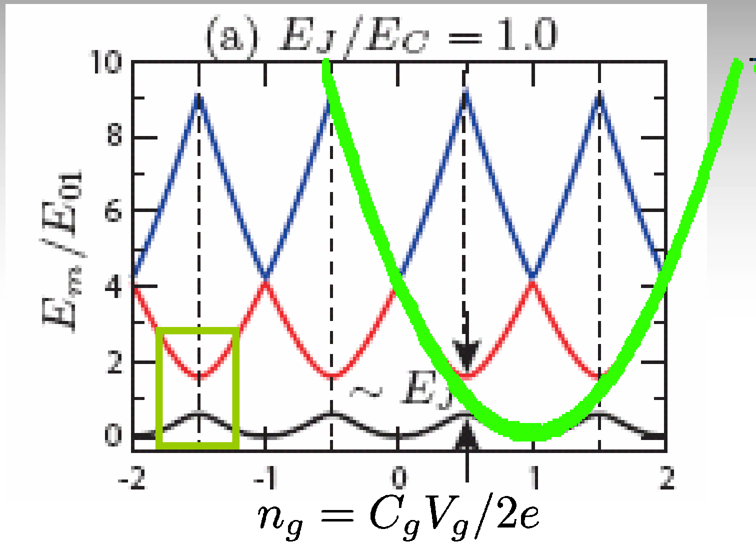
exact solution with Mathieu functions

$$\Psi_m(\varphi) = \frac{1}{\sqrt{2}}\mathrm{me}_{-2m}\left(-\frac{E_J}{2E_C}, \frac{\varphi}{2}\right)$$

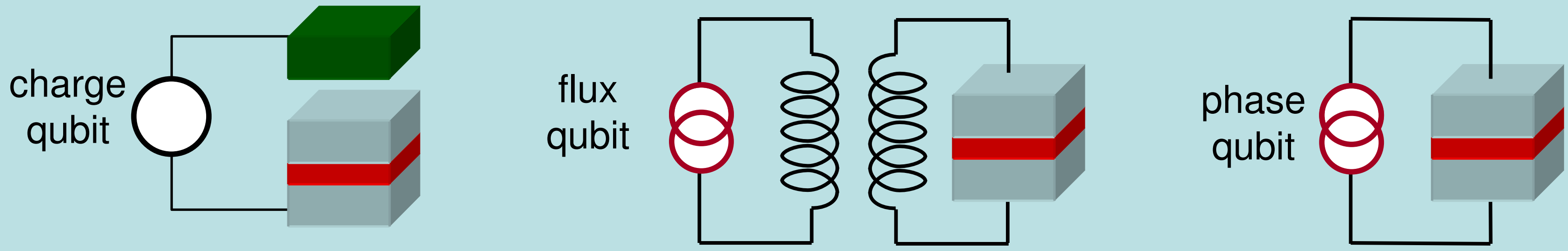
Cooper pair box: SC qubit

CPB: first example for SC qubit!





Different types of SC qubits: (we will mainly consider the CPB)

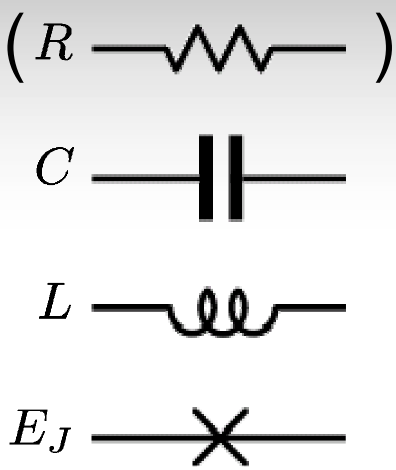


Reviews:

Yu. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001)
M. H. Devoret, A. Wallraff and J. M. Martinis, *cond-mat/0411172* (2004)
J. Q.You and F. Nori, Phys. Today, Nov. 2005, 42

Building Quantum Electrical Circuits

circuit elements

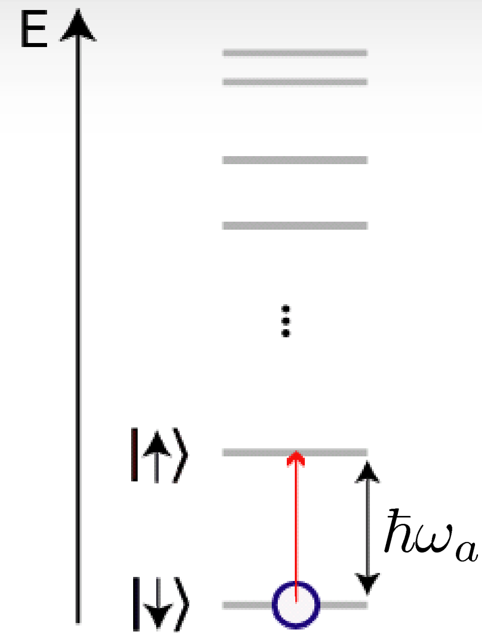


ingredients:

- nonlinearities
- low temperatures
- small dissipation
- isolation from environment

M. H. Devoret, *Quantum Fluctuations* (Les Houches Session LXIII), Elsevier 1997, pp. 351–386.

SC qubits:
macroscopic artifical atoms



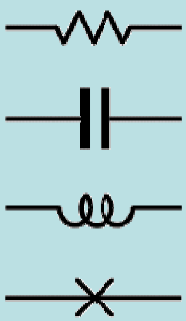
$$\hat{H} = \frac{\hbar\omega_a}{2}\hat{\sigma}^z$$

Two-level system: fake spin 1/2

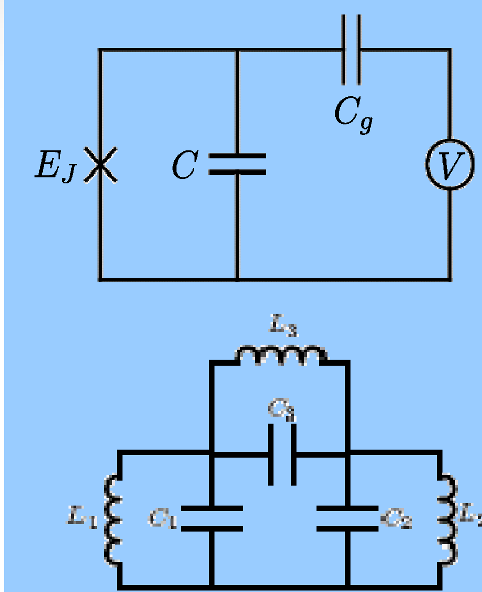
Circuit quantization

(see exercise)

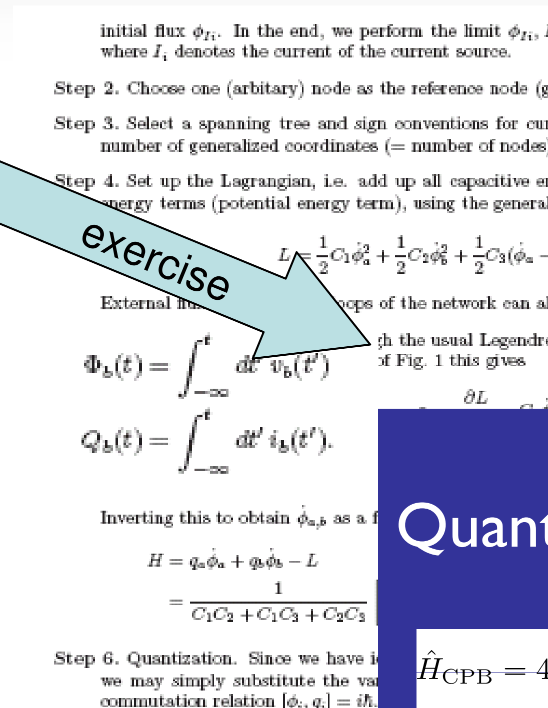
circuit elements



circuits



exercise


initial flux ϕ_{Ii}. In the end, we perform the limit ϕ_{Ii}, L
where I_i denotes the current of the current source.

Step 2. Choose one (arbitary) node as the reference node (gr

Step 3. Select a spanning tree and sign conventions for curr
number of generalized coordinates (= number of nodes)

Step 4. Set up the Lagrangian, i.e. add up all capacitive en
energy terms (potential energy term), using the generali

$$L = \frac{1}{2}C_1\dot\phi_a^2 + \frac{1}{2}C_2\dot\phi_b^2 + \frac{1}{2}C_3(\dot\phi_a -$$

External ... loops of the network can als

... h the usual Legendre
... of Fig. 1 this gives

$$\Phi_b(t) = \int_{-\infty}^{t} dt'\ v_b(t')$$

$$Q_b(t) = \int_{-\infty}^{t} dt'\ i_b(t').$$

Inverting this to obtain $\dot\phi_{a,b}$ as a f

$$H = q_a\dot\phi_a + q_b\dot\phi_b - L$$

$$= \frac{1}{C_1C_2 + C_1C_3 + C_2C_3}$$

Step 6. Quantization. Since we have i
we may simply substitute the va
commutation relation $[\phi_i, q_i] = i\hbar$.

Quantum Hamiltonian

$$\hat H_{\mathrm{CPB}} = 4E_C\hat n^2 - \frac{E_J}{2}\sum_{n=-\infty}^{\infty}\Big[|n+1\rangle\langle n| +$$

► M. H. Devoret, *Quantum Fluctuations*
(Les Houches Session LXIII), Elsevier 1997, pp. 351–386.