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# Level spacing distribution of scissors mode states in heavy deformed nuclei ☆

J. Enders [a,1], T. Guhr [a,b], N. Huxel [a,2], P. von Neumann-Cosel [a], C. Rangacharyulu [a,c], A. Richter [a]

[a] *Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*
[b] *Max-Planck-Insititut für Kernphysik, D-69117 Heidelberg, Germany*
[c] *Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, SK S7N 5E2, Canada*

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**Abstract**

We analyse the level spacing distributions of scissors mode states in heavy deformed nuclei. To this end, we use an ensemble of photon scattering data of 13 nuclei. The results are consistent with a Poissonian behaviour which indicates that the correlations between the states are weak only. The significance of this finding is critically investigated. With numerical simulations we study the possible influence of detection thresholds in the photon scattering experiments. A similar analysis of other dipole modes suggests that the collective nature shown for the scissors mode may be a general phenomenon of low-lying states in deformed nuclei. © 2000 Elsevier Science B.V. All rights reserved.

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The complexity of the nuclear many body problem led Wigner, more than forty years ago, to the introduction of Random Matrix Theory (RMT), a review of which can be found in Ref. [1]. This statistical approach models spectral fluctuation properties: If the levels are correlated, one expects a linear repulsion between them and Dyson statistics for the nearest neighbour spacing distribution (NNSD). However, if correlations are absent, there is no level repulsion and the NNSD is of Poisson-type, i.e. an exponential distribution. The validity of this ansatz was confirmed in various data analyses, see Ref. [1]. Haq et al. [2] analysed the nuclear data ensemble, comprising of s-wave neutron capture resonances in heavy nuclei, populating levels of spin $J = 1/2$ at an excitation energy $E_x \approx 8$ MeV. Good agreement with the Wigner–Dyson behaviour was found. Shriner et al. [3] analysed spacing distributions in different mass regions starting at low excitation energies. Wigner–Dyson statistics was found in

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*E-mail address:* vnc@ikp.tu-darmstadt.de (P. von Neumann-Cosel).

[1] Present address: National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing (MI), USA.

[2] Deceased.

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most regions. However, they observed a tendency towards a Poisson behaviour in the regions where the deformation is large. This picture was strongly supported by Garrett et al. [4] who showed that the NNSD of an ensemble of states in rare earth nuclei with larger total spin $J$, but near the yrast line, is Poissonian.

Thus, the picture emerges that, in heavy nuclei, high-lying single-particle states show Wigner–Dyson statistics, whereas low-lying collective states lack correlations and yield a Poisson distribution. This, in turn, allows us to use RMT to conclude from spectral statistics if excitations are mainly of collective or of single-particle character.

In this Letter, we apply this idea to the states which belong to the scissors mode. This low-lying isovector magnetic orbital dipole mode was first discovered in electron scattering experiments in Darmstadt in the early 1980's [5] and has been subject of intense experimental and theoretical investigations [6]. An unprecedented data set is available now from extensive nuclear resonance fluorescence (NRF) experiments [7,8] covering doubly even nuclei in the $N = 82$–$126$ major shell. By combining the data sets from different nuclei, one can extract statistically significant results. We notice that our study is different from the ones discussed above because we have an ensemble of excitations belonging to the same mode which is confined to a certain energy window of about $2.5 < E_x < 4.0$ MeV. This energy interval was chosen based on the arguments presented in Refs. [8,9]. At lower energies two-quasiparticle excitations with $J^\pi = 1^+$ have been observed [10], while at higher excitation energies spin contributions dominate the magnetic dipole response [11]. Thus, we extend the study of nuclear dynamics into a regime of low spin with excitation energies of a few MeV only, but well above the yrast line. It has been much discussed whether or not the scissors mode is collective [12]. Following the line of reasoning above, our present study sheds new light on this issue.

The selection of the scissors mode states from the set of dipole transitions excited in NRF experiments is indicated schematically by the inlet of Fig. 1. States with $J^\pi = 1^+$ are resonantly excited through photons from a continuous bremsstrahlung spectrum (dashed arrow). In deformed nuclei, decays to the g.s. and the first excited $2^+$ state are observed (full arrows). The Alaga rules are applied to determine the parities from their branching ratios. They allow to deduce the $K$ quantum number of the excited state. Parity assignments using Compton polarimeters [7] support the identification of $\Delta K = 1$ transitions with the depopulation of $1^+$ states, whereas the decay of $1^-$ states is characterised by $\Delta K = 0$. By way of example, the resulting $B(\mathrm{M1})$ strength distribution of g.s. transitions to the scissors mode states for $^{154}$Sm is shown in Fig. 1. Levels without decay branches to the $2_1^+$ state have been analysed separately since their structure remains unclear [8].

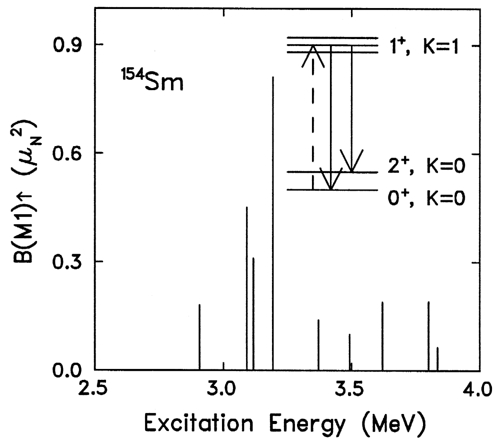


Fig. 1. Strength distribution of $B(\mathrm{M1})$ transitions from the g.s. to the scissors mode states deduced from NRF experiments for the example of $^{154}$Sm [14]. Inlet: Scheme of identification of scissors mode states, see text.

In order to avoid effects due to very short level sequences, only nuclei with at least 8 levels in the given energy interval have been considered for the analysis. A data ensemble has been constructed with a total number of 152 states from data of 13 heavy deformed nuclei ($^{146,148,150}$Nd [13], $^{152,154}$Sm [14], $^{156,158}$Gd [15], $^{164}$Dy [16], $^{166,168}$Er [17], $^{174}$Yb [18], $^{178,180}$Hf [19]). In addition, it was possible to form two other data ensembles for the $J^\pi;K = 1^-;0$ states (74 from $^{150}$Sm [14], $^{164}$Dy [16], $^{166,168}$Er [17], $^{172}$Yb [18], and $^{178,180}$Hf [19]) and the dipole excitations without decay branch to the $2_1^+$ state (106 from $^{146,148,150}$Nd [13], $^{150}$Sm [14], $^{164}$Dy [16], $^{168}$Er [17], $^{180}$Hf [19]). For the latter two ensembles, the energy region 2.0–4.0 MeV was investigated.

For the statistical analysis the spectra have first to be unfolded, i.e. the energy dependence of the average level spacing has to be removed [1,20]. In general, one expects in nuclei a smooth, monotonic increase of the average level density which can be parametrized by models like back-shifted Fermi gas or the constant temperature approach [21]. In case of the scissors mode, however, we deal with an excitation mechanism which is limited to a certain energy window. Thus, the density of only those states will be a function with a local maximum. Indeed, experimental evidence for such a behaviour of the average level spacing was found in the analysis of primary $\gamma$ rays in heavy deformed nuclei [22]. Hence, instead of a back-shifted Fermi gas or constant temperature model we chose a polynomial of degree three. Our empirical polynomials give a much better description of the experimental cumulative level density than the back-shifted Fermi gas model. We emphasize that a good agreement usually biases the results towards a Wigner–Dyson behaviour. Since we eventually find a clear tendency towards Poisson, we are convinced that it is not caused by our polynomial unfolding procedure.

From the unfolded spectra, the nearest-neighbour spacing distribution (NNSD), the cumulative NNSD, the number variance $\Sigma^2$ and the spectral rigidity $\Delta_3$ were extracted from the data ensembles [1,20]. The result for the states attributed to the scissors mode ($J^{\pi};K=1^{+};1$) is shown in Fig. 2: All evaluated statistical measures agree remarkably well with the Poissonian behaviour of uncorrelated levels. For comparison, the Poisson and the Wigner–Dyson distributions are also displayed. Although the individual level sequences are rather short, the functions $\Sigma^2$ and $\Delta_3$ clearly show the lack of long-range correlations. However, one should keep in mind that the typical length of a sequence from one single nucleus is of the order of ten only.

A critical discussion of this remarkable finding is called for. Obviously, the analysis is only statistically significant if the sequences are complete. The missing of levels can strongly influence the resulting distributions. In photon scattering experiments, an observation limit exists. Weakly excited states escape detection because of noise or background in the spectra. It is therefore important to study the influence of the experimental detection threshold on the statistical analysis. It could so happen that the level density of scissors states is much higher than the actual number of detected excitations. This experimental limitation would then affect the level spacing distribution. Especially, an underlying Wigner–Dyson behaviour could be masked by that effect.

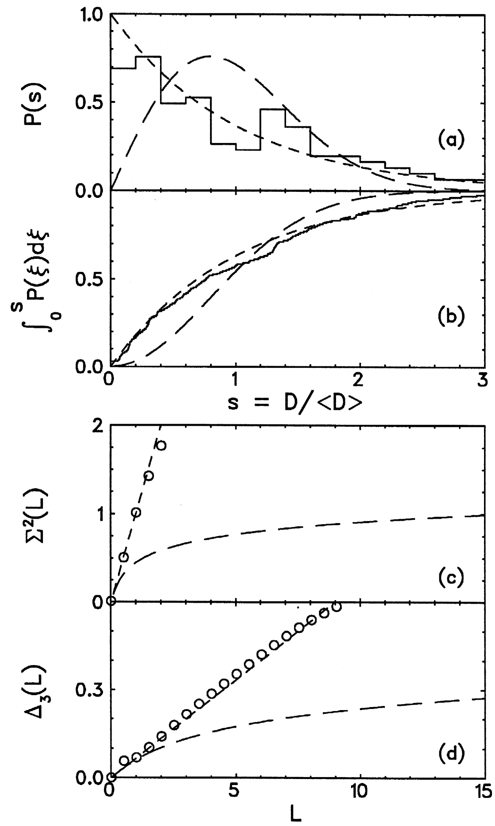


Fig. 2. Level spacing distributions of scissors mode states in heavy deformed nuclei (152 states from 13 nuclei). Shown are (a) the nearest-neighbour spacing distribution, (b) the cumulative nearest-neighbour spacing distribution, (c) the number variance $\Sigma^2(L)$ and (d) the spectral rigidity $\Delta_3(L)$. Full histograms and open circles display the data. Poissonian behaviour and expectations from the Gaussian Orthogonal Ensemble (Wigner) are shown as short and long dashed lines, respectively.

To investigate this we numerically generated a set of levels whose NNSD is of Wigner–Dyson type. To each state a transition intensity $I$ was assigned drawn from a Porter–Thomas distribution which corresponds to Wigner–Dyson level statistics [23]. Levels below a given threshold were then removed from the ensemble, and the spacing distribution was reanalysed thereafter. Typical results are shown in Fig. 3(a)–(c) for cut-offs $I > 0.2\langle I\rangle$, $0.5\langle I\rangle$ and $1.0\langle I\rangle$, respectively, where $\langle I\rangle$ denotes the average

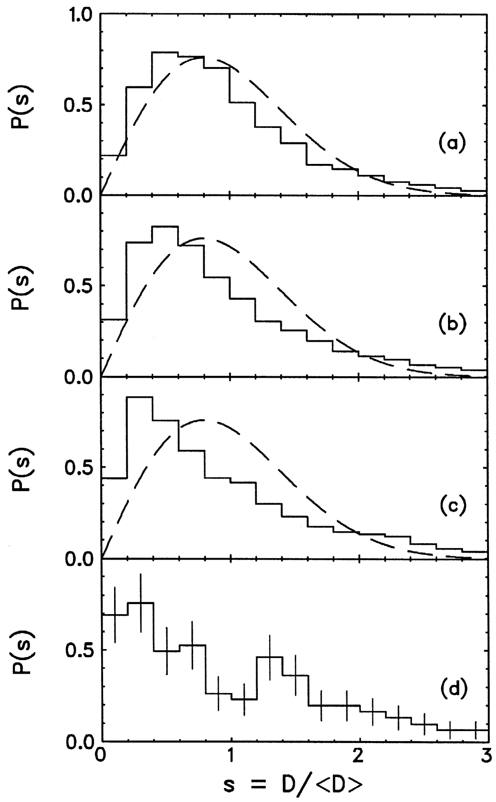


Fig. 3. Influence of a detection threshold on a GOE data set. For the assumption of a Porter–Thomas intensity distribution different cut-offs are shown: (a) $I > 0.2\langle I \rangle$, (b) $I > 0.5\langle I \rangle$, and (c) $I > 1.0\langle I \rangle$. The dashed line shows a Wigner distribution from which the analysis was started. Part (d) shows the NNSD which was extracted from the scissors mode states. In part (d), statistical error bars are given.

intensity of the initial distribution. For these thresholds, a fraction of 71%, 50% or 33% of the original number of states would be observed in an experiment. For comparison, the NNSD extracted from the data of the scissors mode states is shown in Fig. 3(d). The vertical lines show the statistical errors $(1\ \sigma)$ from the number of events in each bin. However, these errors are highly correlated and cannot be used directly to conclude on possible deviations of the shape of the distribution.

To judge the significance of our analysis we need an estimate for the number of states that could have been missed in the experiment. Recently, a parameter-free description of the scissors mode in heavy nuclei has been derived using a sum-rule approach [8]. From this model-independent analysis we infer that the non energy-weighted sum rule for the scissors mode strength at low excitation energies is typically exhausted on a 90% level. For a Porter–Thomas distribution of the excitation strengths, those 10% of strength which might have escaped detection would correspond to a threshold $I > 0.5\langle I \rangle$. We emphasize that, strictly speaking, this applies to correlated levels only. In an uncorrelated situation, the distribution of the wave function components is very different from a Porter–Thomas distribution. However, deduced quantities such as partial widths or transition matrix elements should show a Porter–Thomas-like distribution as well.

The simulated distribution corresponding to this estimate is given in Fig. 3(b). Clearly, the comparison shows that the original spacing distribution [cf. Fig. 3(d)] must be much closer to a Poisson than to a Wigner–Dyson distribution. Moreover, we notice that the probability $P(s)$ of small spacings contained in the first bin of our experimentally obtained distribution is significantly higher than found in all simulations. This strongly points to a lack of correlations in the experimental level distributions. Consequently, we may draw the remarkable conclusion from our analysis that the scissors mode states all have the same structure and are excited collectively by the same mode. Besides the effect of levels missed in the analysis, the determination of the average level density might lead to additional uncertainties.

The dominantly collective behaviour supported by our analysis is consistent with the extreme semiclassical picture, in which the two ellipsoids of all neutrons and all protons rotate against each other (‘‘scissors’’). Nevertheless, since one observes in the experiment sequences of states grouped around the energy predicted in this extreme picture, the latter cannot fully account for the physical excitation mechanism. Importantly, the very fact that this group of states is confined to a certain energy window implies that they are somehow correlated. The analysis above, however, shows that these correlations are weak. Conventionally, one would interpret such a group of states as generated by a fragmentation of the semiclassical state which acts as a doorway [24]. A set of many-particle many-hole states having Wigner–Dyson statistics couples to the doorway [25].

However, the spreading width, i.e. the mixing matrix elements of the fragmentation process consistent with the extracted NNSD, is much smaller than the values of about 20–100 keV estimated from the approach [26] underlying the sum rule analysis of the scissors mode [8]. This indicates that the levels of the scissors mode form a set of states with similar structure which are excited by a common mechanism. In other words, in view of the weakness of their correlations we find it doubtful to distinguish one of these states as a doorway. We prefer to think of all of them as equal and dominantly collective. This interpretation is furnished by the qualitative observation that the strength distributions are irregular and not Lorentzian-shaped as one would expect in a doorway picture.

To acquire further insight into the nature of excitations in this energy region, we also analysed the NNSD for the 74 states with $J^{\pi};K = 1^{-};0$ and for the 106 states with $J^{\pi} = 1^{\pm}$ where no branching to the first excited state could be observed. The extracted distributions are displayed in Fig. 4(a) for the $1^{-}$ states and (b) for the states without decay branch to the $2_1^+$ level. For these modes we find a tendency towards Poisson behaviour as well, although less pronounced than in the case of the scissors mode. However, in case (b) because of the missing parity

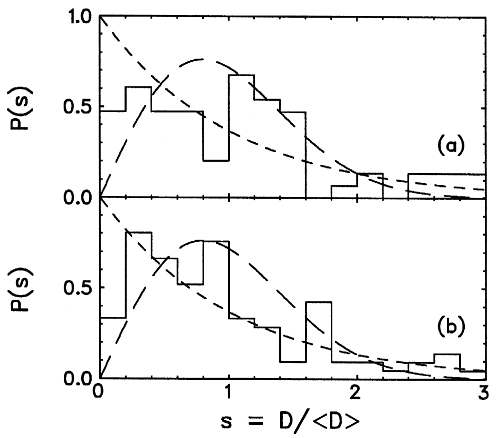


Fig. 4. Nearest-neighbour spacing distributions (a) of the $J^{\pi};K = 1^{-};0$ states and (b) of the dipole excitations without decay branch to the $2_1^+$ state. The histograms represent the data; Poissonian behaviour and the GOE expectation are shown as short and long dashed lines, respectively.

information the result may be partially faked by the superposition of distributions with different quantum numbers. For both ensembles the possible influence of missing levels cannot be quantified. Similar to the scissors mode, the large value of $P(s)$ at low $s$ observed in the NNSD of the $J^{\pi};K = 1^{-};0$ states largely excludes strong correlations despite the fact that there is no theoretical guide to the total E1 strength which may be expected in the investigated energy region.

In summary, we have analysed the level spacing distribution of scissors mode states in heavy deformed nuclei from thirteen photon scattering spectra forming a data ensemble consisting of 152 states. The results show strong evidence for uncorrelated Poisson behaviour. The influence of missing levels on this result due to the experimental conditions was demonstrated to be negligible. Thus, the levels of the scissors mode which are characterised by identical quantum numbers ($J^{\pi};K = 1^{+};1$) form a set of states excited by a common mechanism. This gives independent evidence for the collective nature of the scissors mode.

We find indications for a similar behaviour of other dipole excitations in this energy and mass region. This seems to suggest that most, if not all, low-lying excitations in these well deformed nuclei possess this collective feature. However, at present we admit that the physical picture of a possible collective mechanism exciting these states remains unclear. It is certainly of interest to investigate other low-lying dipole modes like the so-called ‘pygmy resonance’, i.e. the E1 strength concentration below the particle emission threshold in heavy spherical nuclei [27]. Studies are underway where a detailed data set is established from measurements of the E1 fine structure in nuclear resonance fluorescence experiments [28].


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