Lecture 2: hadron production and the QCD phase boundary

- Comments on the QCD phase transition
- Hadron production and the chemical freeze-out curve
  - hadron yields and the statistical model
  - hadron yields and the phase boundary
  - interpretation:
    - 2-body collisions don't equilibrate
    - the phase transition drives equilibration through multi-hadron collisions
    - Hagedorn states as possible intermediaries
  - Speculation about the phase boundary at large $\mu$
- Open charm and Charmonia
- Outlook
Critical energy density and critical temperature

$T_c = 173 \pm 12 \text{ MeV}$

$\varepsilon_c = 700 \pm 200 \text{ MeV/fm}^3$

for the (2 + 1) flavor case: the phase transition to the QGP and its parameters are quantitative predictions of QCD.

The order of the transition is not yet definitively determined.

Lattice QCD calculations for $\mu_B = 0$

Karsch et al, hep-lat/0305025
The QCD phase boundary – recent results from lattice QCD

S. Ejiri et al, hep-lat/0312006

Note: $3 \mu_q = \mu_B$

Tri-critical point not (yet) well determined theoretically
From AGS energy on, all hadron yields in central PbPb collisions reflect grand-canonical equilibration.

Strangeness suppression observed in elementary collisions is lifted.

For a recent review see:

pbm, Stachel, Redlich, QGP3, R. Hwa, editor, Singapore 2004, nucl-th/0304013
Thermal model description of hadron yields

Grand Canonical Ensemble

\[ \ln Z_i = \frac{V g_i}{2 \pi^2} \int_0^\infty \frac{p^2 dp}{\exp((E_i - \mu_i)/T)} \ln(1 \pm \exp(-(E_i - \mu_i)/T)) \]

\[ n_i = N/V = -\frac{T \partial \ln Z_i}{V \partial \mu_i} = \frac{g_i}{2 \pi^2} \int_0^\infty \frac{p^2 dp}{\exp((E_i - \mu_i)/T)} \pm 1 \]

\[ \mu_i = \mu_B B_i + \mu_S S_i + \mu_I I_i^3 \]

for every conserved quantum number there is a chemical potential \( \mu_i \) but can use conservation laws to constrain:

- Baryon number: \( V \sum_i n_i B_i = Z + N \) \( \rightarrow V \)
- Strangeness: \( V \sum_i n_i S_i = 0 \) \( \rightarrow \mu_S \)
- Charge: \( V \sum_i n_i I_i^3 = \frac{Z - N}{2} \) \( \rightarrow \mu_I \)

This leaves only \( \mu_b \) and \( T \) as free parameter when \( 4\pi \) considered for rapidity slice fix volume e.g. by \( dN_{ch}/dy \)
the resonance gas partition function contains a sum over all hadronic states

Excellent agreement below $T_c$! Resonance gas approximates QCD

comparison between baryonic pressure from LQCD and from hadron resonance gas

K. Redlich, hep-ph/0406250 and refs. there
Hadro-chemistry at RHIC -- weakly decaying particles

All data in excellent agreement with thermal model predictions

chemical freeze-out at: $T = 175 \pm 8$ MeV

fit uses vacuum masses

new results from SQM04 at Cape Town consolidate this picture

Data at 40 GeV/u Pb+Pb central collisions
\( T = 148 \) MeV,
\( \mu_b = 400 \) MeV

analysis from
pbm, Stachel, Redlich,
nucl-th/0304013
“Quark-gluon plasma 3, p. 491 – 599”
Establishing the chemical freeze-out curve

The first plot: pbm, Stachel

The full curve:
pbm, Stachel, QM1997
chem. freeze-out at constant total baryon density

chem. freeze-out at constant energy/particle

Note: for $\mu < 300$ MeV, LQCD phase boundary coincides with freeze-out curve
Open Issue: the NA49 „horn“ in K/π

The structure near $\sqrt{s} = 8$ GeV is not reproduced but note: natural „smearing“ is $\approx 3$ GeV near $\sqrt{s} = 8$ GeV

Strangeness undersaturated at 80 and 160 A GeV, saturated at all other energies?
excitation functions and thermal model predictions

\[ \frac{K^+}{\pi} \] vs. \( \sqrt{s_{NN}} \) (GeV)

\[ \frac{A}{\pi} \] vs. \( \sqrt{s_{NN}} \) (GeV)

\[ \frac{\bar{n}}{\pi} \] vs. \( \sqrt{s_{NN}} \) (GeV)

\[ \text{Yield} / \Lambda \] vs. \( \sqrt{s_{NN}} \) (GeV)

Peter Braun-Munzinger
Strangeness equilibration at RHIC energies

- Strangeness fully saturated

- Freeze-out points are very close to phase boundary

- Deal with multi-strange baryons
Chemical Equilibration must take place in the Hadronic Phase

- Hadron yields determined by Boltzmann factors with 'free' vacuum masses.
- Particle distribution in QGP phase has no 'memory' of vacuum hadron masses.
- Relative yields are not determined by the strange quark mass but by individual strange hadron masses (at fixed T and m).
- But: the number of strange quarks is determined in the QGP phase! Equilibrium then implies redistribution of strange quarks.
How is chemical equilibration achieved?  
Our Scenario

- Strangeness saturation takes place in the QGP phase.
- Phase transition is crossed from above.
- Near $T_c$ new dynamics associated with collective excitations will take place and trigger the transition.
- Propagation and scattering of these collective excitations is expressed in the form of multi-hadron scattering. Near $T_c$ multi-hadron processes will therefore be dominant. Chemical equilibrium is reached via these multi-hadron scattering events.
Chemical freeze-out takes place at $T_c$!

- Two-body collisions are not sufficient to bring multi-strange baryons into equilibrium.
- The density of particles varies rapidly with $T$ near the phase transition.
- Multi-particle collisions are strongly enhanced at high density and lead to chem. equilibrium very near to $T_c$.

pbm, J. Stachel, C. Wetterich

nucl-th/0311005

Lattice QCD calcs.
By F. Karsch et al.
Evaluation of multi-strange baryon yield

consider situation at $T_{ch} = 176$ MeV first

- rate of change of density for $n_{in}$ ingoing and $n_{out}$ outgoing particles

$$ r(n_{in}, n_{out}) = \tilde{n}(T)^{n_{in}} |\mathcal{M}|^2 \phi $$

with

$$ \phi = \prod_{k=1}^{n_{out}} \left( \int \frac{d^3 p_k}{(2\pi)^3 (2E_k)^4} \right)^{(2\pi)^4} \delta^4 \left( \sum_k p_k^\mu \right) $$

- The phase space factor $\phi$ depends on $\sqrt{s}$ and needs to be weighted by the probability $f(s)$ that multiparticle scattering occurs at a given value of $\sqrt{s}$

- evaluate numerically in Monte-Carlo using thermal momentum distribution

- typical reaction: $\Omega + \bar{N} \rightarrow 2\pi + 3K$

  assume cross section equal to measured value for $p + \bar{p} \rightarrow 5\pi$

  relevant $\sqrt{s} = 3.25$ GeV $\rightarrow \sigma = 6.4$ mb

- compute matrix element and use for rate of $2\pi + 3K \rightarrow \Omega + \bar{N}$
reaction $2\pi + 3K \rightarrow \Omega + \bar{N}$ leads to

$$r_\Omega = 0.00014 \text{ fm}^{-4} \text{ or } r_\Omega/n_\Omega = 1/\tau_\Omega = 0.46/\text{fm}$$

⇒ can achieve final density starting from 0 in 2.2 fm/c!

similarly one obtains

for $3\pi + 2K \rightarrow \Xi + \bar{N}$ \hspace{1cm} \tau_\Xi = 0.71 \text{ fm/c}$

and

for $4\pi + K \rightarrow \Lambda + \bar{N}$ \hspace{1cm} \tau_\Lambda = 0.66 \text{ fm/c}$

Peter Braun-Munzinger
Density dependence of characteristic time for strange baryon production

- Near phase transition particle density varies rapidly with $T$.
- For small $\mu_b$, reactions such as $KKK\pi\pi \rightarrow \Omega N_{\text{bar}}$ bring multi-strange baryons close to equilibrium.
- Equilibration time $\tau \propto T^{-60}$!
- All particles freeze out within a very narrow temperature window.

p bm, J. Stachel, C. Wetterich
nucl-th/0311005
Recent work by C. Greiner, H. Stoecker et al. hep-ph/0412 095, following up on our approach:

- multi-hadron collisions are channeled through heavy (~ 1-2 GeV) Hagedorn doorway states
- detailed balance is applied through-out
- decay of the Hagedorn states leads to rapid production of (multi-strange) baryons
- nucleon production less problematic

As in our approach, multi-particle plasma correlations near $T_{c}$ lead to complete strangeness saturation. Chemical freeze-out takes place at the phase boundary.
What about pp and e+e- collisions?

- Thermal fits describe hadron yields with $T \sim 160$ MeV

- Hadronization may be pre-thermalization process

- But: multi-strange baryons can only be reproduced by ad-hoc strangeness suppression factor implying incomplete equilibration
Suppression factor of 2 implies Omega baryons are factor 8 off the equilibrium value

Suppression is not due to canonical thermodynamics (phi problem, K. Redlich)

Multi-meson fusion not effective since no high density phase

'Temperature' in pp and e+e- reflects hadronization but not phase transition.

The existence of a medium in AA collisions also leads to the result that T is not universal (at $T = 160$ MeV as in e+e- and pp) but varies with $\mu$: $T=140$ MeV at $\mu = 400$ MeV, e.g.
Analysis of pp collisions


pp data, $\sqrt{s} = 27.6$ GeV

canonical (volume) suppression vs $\gamma_S$ factor (non-equilibrium), $T = 165$ MeV

Analysis by K. Redlich, see pbm, Stachel, Redlich, nucl-th/0304013

$\gamma_S$ factor needed to describe $\phi$ production

Observed strangeness suppression is **not** described by equilibrium thermodynamics
What about lower beam energies?

- at top SPS energy numbers work out nearly the same as at RHIC
- at 40 A GeV/c pion and kaon densities lower by 1/3 $\rightarrow n_\Omega$ increases by factor 12
- but: other reactions involving baryons must come into play at high baryon density:
  $N\rho KK \rightarrow \Omega\pi$ or $N\pi\pi KK \rightarrow \Omega\rho$
The QCD phase diagram and chemical freeze-out

Data are nearly described by curve of constant critical energy density

Conjecture: chemical freeze-out points delineate the QCD phase boundary also at larger $\mu$ down to AGS energy
A remark on critical energy density

- Along the Fodor-Katz phase boundary, critical energy density increases with increasing $\mu$
- At $\mu = 0$, $\varepsilon_{\text{crit}} = 0.6$ GeV/fm$^3$
- At $T = 160$ MeV and $\mu = 650$ MeV, $\varepsilon_{\text{crit}} \approx 2.7$ GeV/fm$^3$
  
  calc. within hadron resonance gas model, no excluded volume correction
- There are 1.46 baryons/fm$^3$ and 0.44 mesons/fm$^3$ at this point

Phase boundary at $\mu = 650$ MeV is very likely at lower $T$
Is heavy quark production also statistical?

The $\psi'/\psi$ ratio approaches thermal value

Boltzmann factor suppresses thermal production of charm quarks: $m_c/T >> 1$

Thermal production is negligible, even at LHC energy

Charm quarks must be produced in hard collisions

newest NA50 data confirm this, see also: Kostyuk&Stoecker, hep-ph/0501077
Quarkonia Production through Statistical Recombination

- Heavy Quark Production in Hard Scattering Only
- Heavy Meson and Quarkonia Production at Phase Boundary with Statistical Weights from a Thermalized Hadron Gas
- Expect Large Quarkonia Enhancement (Compared to Production in Hard Collisions) at Collider Energies (esp. LHC)
- Sensitive Test for Deconfinement: Recombination Assumes Free Travel over Full System and Thermal Equilibrium for Heavy Quarks


suggested mechanism similar to production of multi-strange baryons, (pbm, stachel, wetterich, nucl-th/0311005)
typical reaction: $\text{DD}_{\text{bar}} + \pi\pi\pi \Rightarrow J/\psi + \pi$

note: $\text{DD}_{\text{bar}}^* \Rightarrow J/\psi + \pi$ provides already about 10% of $J/\psi$ at RHIC energy, see, e.g., pbm, K. Redlich, Eur. J. Phys. C16 (2000) 579
Thermal equilibrium for charm quarks?

- for statistical hadronization to work, charm quarks must approach **thermal** equilibrium.
- recent insight: charm quarks loose energy in the hot medium both by gluon radiation and by scattering: no strong 'dead-cone' effect
- charm quarks should exhibit anisotropic flow and energy loss

both Phenix and Star show first indications of charm quark flow
Charmonia and open charm

- J/ψ modification in the QGP Satz&Matsui 1985
- Measure charmonia relative to open charm reference
- Simultaneous measurement of J/ψ and open charm to come from RHIC and NA60@SPS
- At RHIC energy and above, $N_{cc} \gg 1 \rightarrow$ charmonia can be formed by recombination, $N_{J/ψ} \propto N_{cc}^2$
- At LHC energy, enhancement fingerprint of deconfinement

All charmed hadrons are also formed at the phase boundary!!
Comparison to first RHIC Data

Note centrality dependence

Need much improved RHIC data, to come from Run 4

Calculations: recombination model
LHC data should exhibit large enhancement!
charmonium data – newest results from QM2005

strong suppression not observed

Energy dependence of $J/\psi$ production

$J/\psi$ Excitation Function

Pb–Pb (Au–Au) central collisions

$\rightarrow$ Transition from Suppression to Enhancement

$\rightarrow$ more precise data needed!

RHIC Energy in Balance Region

LHC Energy: Enhancement as Fingerprint of Deconfinement

At RHIC and LHC, $n_{cc} >> 1 \rightarrow$ recombination!!
Chemical equilibration of multi-strange hadrons is obtained through multi-hadron collisions near (during) the phase transition.

Chemical freeze-out at RHIC and top SPS energy coincides with phase boundary from LQCD.

Experimental determination of critical temperature:
\[ T_c = T_{\text{chem}} + 8 \pm 18 \text{ MeV} \]

Charm and charmonia produced through stat. hadronization:
test for deconfinement – need data from RHIC and (especially) LHC

Open questions:
- Where is phase boundary at lower energies?
- Is the full chemical freeze-out curve coincident with the phase boundary?
- Are multi-strange hadrons also equilibrated at SIS energy?
- What about LHC energies

Progress in determination of fundamental QCD parameters from nuclear collisions
2-body collisions are not enough

typical densities at $T_{ch}$: $\rho_\pi = 0.174/\text{fm}^3$ (incl. res.) $\rho_K = 0.030/\text{fm}^3$ $\rho_\Omega = 0.0003/\text{fm}^3$

- To maintain equilibrium even for 5 MeV below $T_{ch}$ need relative rate change

$$\frac{|\bar{\Omega}|}{n_\Omega} - \frac{|\bar{K}|}{n_K} = \tau_\Omega^{-1} - \tau_K^{-1} = (1.10 - 0.55)/\text{fm} = 0.55/\text{fm}.$$ 

So, $\Omega$ density needs to change by 100% within 1 fm/c

- Typical reactions with large cross sections of 10 mb and relative velocity of 0.6 give

  $\Omega + \pi \rightarrow \Xi + K$  $\rightarrow$  $\bar{\Omega}/n_\Omega = n_\pi \langle \nu_\tau \sigma \rangle = 0.086/\text{fm}$

  $\pi + \pi \rightarrow K + \bar{K}$ ($\sigma = 3$ mb)  $\rightarrow$  $\bar{K}/n_K = 0.18/\text{fm}$

  i.e. much too slow to maintain equilibrium even over $\Delta T = 5$ MeV!

- Even much more difficult: to produce large $\Omega$ abundance

  assume hadronization like in pp, factor 8 too few $\Omega$s, to produce them within 1 fm/c

  need reactions that provide $\bar{\Omega}/n_\Omega = 1.0$  $\Rightarrow$ not with 2-body reactions

Check numerics via detailed balance

- Initially manifestly nonequilibrium situation - start with practically zero $\Omega$ density
- As equilibrium is approached
  
  rates $3K + 2\pi \rightarrow \Omega + \bar{N}$ and $\Omega + \bar{N} \rightarrow 3K + 2\pi$ have to become equal

- back and forth reactions scale very differently with pion density
  
  $\rightarrow$ only at one density can they be equal

- to explicitly check these rates now use pion, kaon, nucleon densities before strong decays,
  
  i.e. without resonance feeding

  (for all resonances corresponding rates have to be calculated accordingly)

- find: creation of $\Omega$ with $r_\Omega/n_\Omega = 3.4 \times 10^{-3}/\text{fm}$

  and annihilation of $\Omega$ with $r_\Omega/n_\Omega = 1.4 \times 10^{-3}/\text{fm}$

  for equal rates reduce density by 25 %

  reduce $T$ by 2-3 MeV or excluded volume a bit larger
Variation of fireball temperature with time

Values chosen appropriate for RHIC Au + Au collisions

- Assume: \( T_{ch} = 176 \text{ MeV} \)
- Density decrease between chemical and thermal freeze-out: 30 %
- Two-pion correlation data: \( R_{side} = 5.75 \text{ fm}, R_{long} = 7.0 \text{ fm}, \text{ mean } \beta_t = 0.5, \beta_{long} = 1 \)
- Isentropic expansion \( \tau_f = 0.9 - 2.3 \text{ fm}, T_f = 158 - 132 \text{ MeV} \) (uncertainty due to variation in density profile)
- Near \( T_c \): rate of decrease in temperature \( |\dot{T}/T| = \tau^{-1}_T = (13 \pm 1) \% /\text{fm} \)

Peter Braun-Munzinger
What about centrality dependence of chemical equilibration?

- Apparent chemical temperature depends little on centrality.
- The importance of multiple collisions should decrease with decreasing particle density, i.e. lower centrality.
- This is expressed in the data as change in $\gamma_s$.
- Note: $\gamma_s = 0.8$ reduces $\Omega$ yield by factor of 2.
Centrality dependence of $\gamma_s$

Cleymans, Kämpfer, Steinberg, Wheaton, hep-ph/0212335

Fit $\mu_B$ and $\gamma_s$ to $\pi$, $K$, $p$ yields

$f_2$ fraction of $N_{\text{part}}$ with multiple collisions
Centrality dependence of $\gamma_s$

S. Wheaton et al, SQM04, Au+Au analysis, RHIC energy
increasing $N_{\text{part}}$ → increasing particle density
→ chemical equilibration is reached